LPV DETECTION FILTER DESIGN FOR BOEING 747-100/200

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Abstract

This paper presents a fault detection and isolation (FDI) filter design using a linear parameter varying (LPV) model of the longitudinal dynamics of a Boeing 747 series 100/200. The LPV FDI filter design is based on an extension of the fundamental problem of residual generation concepts elaborated for linear, time-invariant systems. Typically, the FDI filters are designed for open-loop model, and applied in closed-loop. This paper shows the use of the LPV FDI filter for actuator failure in the closed loop LPV system and also for the situation when it is applied to the nonlinear system simulation environment representing the "true" system.

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1 Nomenclature

\( \bar{c} \) 
wing chord, meter

\( c_T \) 
inertia coefficient

\( \bar{d} \) 
dynamic pressure, \( N/m^2 \)

\( S \) 
reference surface area, \( m^2 \)

\( z_{\text{eng}} \) 
engine position z-axis, \( m \)

\( T \) 
trust force, Newton

\( \alpha \) 
angle of attack (AoA), deg

\( \alpha_w \) 
wing design plane, \( \alpha_w = \alpha + 2 \)

\( \sin \alpha \) 
sine of AoA

\( \cos \alpha \) 
cosine of AoA

2 Introduction

Modern control systems and algorithms are becoming more and more complex and sophisticated. Consequently, the issue of availability, reliability, operating safety are of major importance. These issues are important for safety critical systems such as nuclear reactors, cars and aircraft flight control systems. For safety critical systems, the consequence of faults can be extremely serious in terms of human mortality and environmental impact. Therefore, there is a growing need for on-line supervision and fault diagnosis to increase the reliability of safety critical systems.

A traditional approach to fault diagnosis in the wider application context is based on hardware redundancy methods which use multiple sensors, actuators, computers and software to measure and control a particular variable. In analytical redundancy schemes, the resulting difference generated from the consistency checking of different variables is called as a residual signal. The residual should be zero when the system is normal, and should diverge from zero when a fault occurs in the system. This zero and non-zero property of the residual is used to determine whether or not faults have occurred. Analytical redundancy makes use of a mathematical model and the goal is the determination of faults of a system from the comparison of available system measurements with a priori information represented by the mathematical model, through generation of residual quantities and their analysis.
There are various approaches to residual generation for, see e.g. the parity space approach,[8] the multiple model method, detection filter design using geometric approach [14] or on frequency domain concepts,[5] unknown input observer concept,[3] dynamic inversion based detection.[22] Most of the design approaches refer to linear, time-invariant (LTI) systems, but references to some nonlinear cases can be found in reference [3].

The geometric approach to design detection filters was initiated by Massoumi for LTI systems [14] and was used also by Bokor at al., for LTV systems.[4] These concepts have been used to build an linear parameter–varying (LPV) fault detection and isolation (FDI) design procedure in [1] and in reference [2]. Related results for FDI filter design appeared recently for bilinear systems, see Hammouri et al. [9] while Persis and Isidori considered input affine nonlinear systems.[18, 19]

Throughout this paper fault detection and isolation for a class of LPV systems where the state matrix depends affinely on the parameter vector will be considered. This class of systems can be described as:

\[
\begin{align*}
\dot{x}(t) &= A(\rho)x(t) + B(\rho)u(t) + \sum_{j=1}^{m} L_j(\rho)v_j(t) \\
y(t) &= Cx(t),
\end{align*}
\]

(1)

where \(v_j\) are the failures to be detected, \(C\) is right invertible,

\[
A(\rho) = A_0 + \rho_1 A_1 + \cdots + \rho_N A_N,
\]

(2)

\[
B(\rho) = B_0 + \rho_1 B_1 + \cdots + \rho_N B_N,
\]

(3)

\[
L_j(\rho) = L_{j,0} + \rho_1 L_{j,1} + \cdots + \rho_N L_{j,N},
\]

(4)

and \(\rho_i\) are time varying parameters. It is assumed that each parameter \(\rho_i\) and its derivatives \(\dot{\rho}_i\) range between known extremal values \(\rho_i(t) \in [-\overline{\rho}_i, \overline{\rho}_i]\) and \(\dot{\rho}_i(t) \in [-\overline{\dot{\rho}}_i, \overline{\dot{\rho}}_i]\), respectively. Let us denote this parameter set by \(\mathcal{P}\).

The paper is organized as follows. Section 3 gives a very quick review of the fundamental problem of residual generation for LTI systems. Following this, the geometric concepts used will be generalized for the above class of LPV systems. In section 4, the nonlinear and LPV model for the longitudinal motion of the Boeing 747 is presented. The section 5 shows the results of fault detection using simulated LPV and nonlinear models, respectively. The Section 6 provides some concluding remarks.
3 Fundamental problem of residual generation for LTI and LPV systems

Let us consider the following LTI system, that has two failure events.

\[
\dot{x}(t) = Ax(t) + Bu(t) + L_1v_1(t) + L_2v_2(t) \\
y(t) = Cx(t).
\]

(5)

In equation (5), \( x(t) \in \mathcal{X} \) is the state variable, \( u(t) \in \mathcal{U} \) is the known control input, \( y(t) \in \mathcal{Y} \) is the known output, the arbitrary time-varying functions \( v_i(t) \in \mathcal{L}_i \) is the unknown failure modes. The term \( L_1v_1(t) \) represents the faulty behavior of the actuator that we are trying to monitor, i.e., a nonzero \( v_1(t) \) should show up in the output of the residual generator \( r(t) \). Similarly, \( L_2v_2(t) \) represents the faulty behavior of the other actuator which should not affect \( r(t) \). As usual, our observables are the measurement \( y(t) \) and the known actuation signal \( u(t) \). The task to design a residual generator that is sensitive to \( L_1 \) and insensitive to \( L_2 \) is called the fundamental problem of residual generation (FPRG). [15]

Let us denote by \( S^* \) the smallest unobservability subspace (UOS) containing \( \mathcal{L}_2 \), where \( \mathcal{L}_2 = \text{Im}L_2 \). \( S^* \) is the largest UOS in \( \text{Ker}C \) containing \( \mathcal{L}_2 \). The \( S^* \) can be computed by UOSA algorithm:[26]

\[
S_0 = \mathcal{X} \\
S_k = W^* + (A^{-1}S_{k-1}) \cap \text{Ker}C,
\]

(6)

(7)

where \( W^* \) is the minimal \((C,A)\)-invariant subspace containing \( \mathcal{L}_2 \). As it is well known, for LTI models, a subspace \( W \) is \((C,A)\)-invariant if \( A(W \cap \text{Ker}C) \subset W \) that is equivalent with the existence of a matrix \( D \) such that \((A + DC)W \subset W \).

**Proposition 1.** FPRG has a solution if and only if \( S^* \cap \mathcal{L}_1 = 0 \), moreover, if the problem has a solution, the dynamics of the residual generator can be assigned arbitrary.

The equation \( S^* \cap \mathcal{L}_1 = 0 \) indicates that \( m_2 \) should not affect the output of the residual generator \( r(t) \).

Given the residual generator in the form

\[
\dot{w}(t) = Nw(t) - Gy(t) + Fu(t) \\
r(t) = Mw(t) - Hy(t),
\]

(8)
$H$ is a solution of $\text{Ker}HC = \text{Ker}C + S^\ast$, and $M$ is a unique solution of $MP = HC$, where $P$ is the projection $P : \mathcal{X} \to \mathcal{X}/S^\ast$.

In order to obtain the matrices in equation (8), consider $D_0$ such that $(A + D_0C)S^\ast \subset S^\ast$, and denote by $A_0 = A + D_0C|_{\mathcal{X}/S^\ast}$. By construction, the pair $(M, A_0)$ is observable, hence there exists a $D_1$ such that the poles of $N = A_0 + D_1M$ can be assigned arbitrary. Then set $G = PD_0 + D_1H$ and $F = PB$.

Note that the important step in the design of the filter is to place the image of the second failure signature $L_2$ in the unobservable subspace of the residual $r(t)$. Also the necessary condition simply states that the image of the first failure signature $L_1$ should not intersect the unobservable subspace of the residual generator, so that a failure of the first actuator show up in the residual $r$. Moreover, the failure signature $L_1$ is only used to check the solvability condition, and the actual construction of the filter is independent of $L_1$.

FPRG results can be extended to the case of multiple events. This has a solution if and only if $S^\ast_1 \cap \mathcal{L}_i = 0$, where $S^\ast_i$ is the smallest unobservability subspace containing $\tilde{L}_i = \sum_{j \neq i} \mathcal{L}_j$. The block diagram of the extended FPRG (EFPRG) can be seen in Figure 1.

![Block diagram of EFPRG](image)

**Figure 1: Block diagram of EFPRG**

For the parameter varying case one can extend these notions, and introduce the **parameter varying $(C,A)$-invariant subspaces**, as follows:

**Definition 1.** Let $\mathcal{C}(\pi)$ denote $\text{Ker}C(\rho)$. Then a subspace $\mathcal{W}$ is called a parameter varying $(C,A)$-invariant subspace if for all the parameters $\rho \in \mathcal{P}$:

$$A(\rho)(\mathcal{W} \cap \mathcal{C}(\rho)) \subset \mathcal{W}. \quad (9)$$

As in the classical case one has the following characterization of the parameter varying $(C,A)$-invariant subspaces:[2]
Proposition 2. \( W \) is a parameter varying \((C,A)\)-invariant subspace if and only if for any \( \rho \in \mathcal{P} \) there exists a state feedback matrix \( D(\rho) \) such that

\[
(A(\rho) + D(\rho)C(\rho))W \subset W. \tag{10}
\]

The set of all parameter varying \((C,A)\)-invariant subspaces containing a given subspace \( B \), is a lower semilattice with respect to the intersection of subspaces. This semilattice admits a minimum, denoted by

\[
W^*_{p,v}(B) := \min W(C(\rho), A(\rho), B). \tag{11}
\]

The notion of “unobservability subspace” extends for the LPV systems considered in this paper as the largest subspace such that there exist a parameter dependent gain matrix \( D(\rho) \) and constant output mixing map \( H \) such that

\[
(A(\rho) + D(\rho)C)S \subset S, \text{ for all } \rho \in \mathcal{P}, \tag{12}
\]

\[
S \subset \text{Ker}HC. \tag{13}
\]

For the LPV systems (1) one can obtain the following algorithm for the computation of the smallest (parameter varying) unobservability subspace \( S^* \) containing \( W \):

\[
S_0 = W + \text{Ker}C
\]

\[
S_h = W + (\cap_{h=0}^{N} A^{-1}_h S_{h-1}) \cap \text{Ker}C,
\]

Let us recall the fact, see [14, 9], that there exist matrices \( H, D(\rho) \) such that \( S^* \) is a parameter varying \((HC, A(\rho) + D(\rho)C)\)-invariant subspace. Moreover, if one starts with a minimal subspace \( W^* \), given by one of the algorithm presented above, then \( \text{Ker}HC = W^* + \text{Ker}C \) and \( D(\rho) \) is determined by \( W^* \).

Proposition 3. For LPV systems (1) one can design a residual generator of type

\[
\dot{w}(t) = N(\rho)w(t) - G(\rho)g(t) + F(\rho)u(t) \tag{14}
\]

\[
r(t) = Mw(t) - Hy(t), \tag{15}
\]

if and only if the smallest (parameter varying) unobservability subspace \( S^* \) containing \( L_2 \) has \( S^* \cap L_1 = 0 \), where \( L_{i} = \cup_{i=0}^{N} \text{Im}L_{i,j} \).

An outline of computation for matrices of a LPV filter is as follows. Let \( H \) be the solution of \( \text{Ker}HC = \text{Ker}C + S^* \), and \( M \) the unique solution of \( MP = HC \), where \( P \) is the projection \( P : \mathcal{X} \rightarrow \mathcal{X}/S^* \). By the
definition of the unobservability subspaces there is a matrix $D_0(\rho)$ such that $(A(\rho) + D_0(\rho)C)S^* \subset S^*$ holds. Then set $A_0(\rho) = A(\rho) + D_0(\rho)C[\chi/S^*, N(\rho) = A_0(\rho)$ and $F = PB(\rho)$.

In order to obtain a quadratically stable residual generator one can set $N(\rho) = A_0(\rho) + D(\rho)M$ in (14), where $D(\rho) = D_0 + \rho_1D_1 + \cdots \rho_ND_N$ is determined such that the LMI defined as,

$$(A_0(\rho) + D(\rho)M)^TP + P(A_0(\rho) + D(\rho)M) < 0$$

holds for all the corner points of the parameter space with a suitable $D(\rho)$ and $P = P^T > 0$.[1]

4 Longitudinal LPV model of Boeing 747-100/200

The LPV model used for FDI filter design is the Boeing 747 series 100/200. The Boeing 747-100/200 is an intercontinental wide-body transport with four fan jet engines designed to operate from international airports. The nonlinear model for the Boeing 747-100/200 was obtained from reference [13].

The body-axes longitudinal motion of the Boeing 747, not including flexible effects, can be described by the following differential equations

$$\dot{\alpha} = \frac{[-F_z \cdot s_\alpha + F_z \cdot c_\alpha]}{m \cdot V_T} + q$$

$$\dot{q} = c_7 \cdot M_y$$

$$\dot{\theta} = q$$

$$V_T = \frac{1}{m} [F_z \cdot c_\alpha + F_z \cdot s_\alpha]$$

$$\dot{h}_e = V_T \cdot c_\alpha \cdot s_\theta - V_T \cdot s_\alpha \cdot c_\theta = V_T \cdot s_\alpha$$

The longitudinal control is performed through a movable horizontal stabilizer with four elevator segments (inboard and outboard elevators) and the engine thrust. Pitch trim is provided by the horizontal stabilizer $\sigma$, and under normal operation the inboard and outboard elevators move together $\delta_E = \delta_{E_1} = \delta_{E_0}$ (for the purposes of this research it is assumed that they move and fail together).
The body-axes aerodynamic forces and moments are given by

\[
F_z = -\eta S \cdot \left[ C_D \cdot c_a - C_L \cdot s_a \right] + \\
\sum_{i=1}^{4} Tn_i - mg \cdot s_0
\]

\[
F_z = -\eta S \cdot \left[ C_D \cdot s_a + C_L \cdot c_a \right]
\]

\[
-0.0436 \sum_{i=1}^{4} Tn_i + mg \cdot c_0
\]

\[
M_y = \bar{q} S \bar{c} \cdot \left\{ C_m - \frac{1}{\bar{c}} \left[ (C_D \cdot s_a + C_L \cdot c_a) \bar{z}_{cg} \right. \right. \\
\left. \left. - (C_D \cdot c_a - C_L \cdot s_a) \bar{z}_{cg} \right] \right. \\
+ \frac{\bar{c} \bar{\alpha}}{V_T} \left[ C_{m\alpha} - \bar{z}_{cg} \bar{c} \cdot C_{b\alpha} \cdot c_a \right] \right\} + \sum_{i=1}^{4} Tn_i \cdot z\eta_0.
\]

In reference [13] the longitudinal nonlinear model of the Boeing 747-100/200 is simplified by reducing the complexity of the aerodynamic coefficients while still maintaining a high degree of accuracy with respect to the full set of aerodynamic coefficients.

The states of the LPV model are angle of attack \( \alpha \), pitch rate \( q \), true velocity \( V_T \), pitch angle \( \theta \) and altitude, \( h_e \). The output measurements are angle of attack \( \alpha \), pitch rate \( q \), true velocity \( V \), pitch angle \( \theta \) and \( h_e \). The inputs are elevator deflection \( \delta_e \), throttle \( T \), stabilizer deflection \( \delta_s \) and an ideal fictitious input assumed to be always equal to 1. The \( A(\rho) \), \( B(\rho) \) matrices of the LPV model used for LPV filter design depend affinely on nine scheduling parameters, \( \rho \). The scheduling variables to be used in the LPV model are the following: \( \rho = [\bar{q}, \frac{q}{V_T}, \frac{1}{V_T}, \gamma, C_{L_{basic}}(\alpha_0, M) \cdot \frac{2}{V_T}, \frac{\partial C_{L_{basic}}}{\partial \alpha}(h_e, M) \cdot \frac{2}{V_T}, C_{D_{Mach}}(M, C_L') \cdot \bar{q}, \frac{\partial C_{L_{basic}}}{\partial \alpha}(h_e, M) \cdot \bar{q}, C_{m_{basic}}(\alpha, M) \cdot \frac{\partial q}{\partial \alpha}]^T \). This set of scheduling variables is formed by combining aircraft stability derivatives and some physical variables (i.e. dynamic pressure, true airspeed and flight path angle). The term \( C_{L_{basic}} \) is the basic lift coefficient for the rigid airplane at the zero stabilizer angle in free air with the landing gear retracted, \( \frac{\partial C_{L_{basic}}}{\partial \alpha}(h_e, M) \) is the change in basic lift coefficient due to change in elevator, \( C_{D_{Mach}}(M, C_L') \) is the Mach effect on Drag coefficient, \( \frac{\partial C_{L_{basic}}}{\partial \alpha}(h_e, M) \) is the change in basic pitching moment coefficient due to change in elevator, \( C_{m_{basic}}(\alpha, M) \) is the basic pitching moment coefficient for the rigid airplane at the zero stabilizer angle in free air with the landing gear retracted.

The LPV model is not classical in the sense that for control design the number of scheduling variables are usually kept small due to the computational load required by the available LPV control synthesis tools. In the present case LPV FDI design based on concepts of the FPRG, the number of variables did not pose a problem and only the constraints related to the affineness of the system matrices and the need
to accurately describe the nonlinear model with the LPV model presented some difficulties. The former constraint was solved by introducing the fictitious input, while the latter was easily solved by avoiding any simplifications of the nonlinear differential equations (this was possible due to the possibility of using unlimited number of scheduling variables).

In order to validate the LPV model, open loop time responses are obtained and compared to the transient response of the nonlinear model. The simulation time is 50 seconds. Figure 2 and Figure 3 show the open loop time responses of the nonlinear system and the LPV models to a square wave elevator deflection of $-1$ degree applied between $5 - 20$ seconds and then a step command of 1 degree from 30 seconds.

![Graphs showing time responses of LPV and nonlinear models](image)

**Figure 2: Time responses of LPV and nonlinear models**
The flight condition is the same for all the time responses: angle of attack $\alpha = 1.2$ degrees, true airspeed $V_T = 241$ m/s, and an altitude of $h_c = 9000$ m. It is observed that time responses of the LPV and the nonlinear models match almost perfectly.

## 5 FDI filter design using LPV model

This section shows the use of a LPV FDI filter to detect an actuator failure in the closed–loop Boeing 747 LPV system and when it is applied to the nonlinear Boeing 747 simulation environment representing the "true" system. In order to implement the FDI filter in closed–loop, a $H_\infty$ LPV controller based on
Total Energy Control System (TECS) concept is used for longitudinal model. The $\mathcal{H}_\infty$ controller used for Boeing model is obtained from reference [6]. The TECS integrates all longitudinal altitude and speed control functions. The TECS achieves consistent decoupled maneuver control for all command modes and flight conditions. The $\mathcal{H}_\infty$ control design objective are to achieve decoupled $\gamma$ and $V$ response of the aircraft, increase $\gamma$ bandwidth independent of engine dynamics and reject disturbances (wind gust, sensor noise). In case of closed-loop, the filter takes the outputs of the aircraft and the controller outputs which is the elevator deflection $\delta_e$ and the throttle $T$.

The LPV FDI filter designed for the Boeing 747/100-200 is sensitive to elevator and throttle failure. The LPV model including elevator and throttle failure can be described as:

\[
\begin{align*}
\dot{x}(t) &= A(p)x(t) + B(p)u(t) + L_{cl}(p)v_{cl}(t) + LT \nu_T(t) \\
y(t) &= Cx(t),
\end{align*}
\] (26)

where

\[
\begin{align*}
A(p) &= A_0 + \rho_1 A_1 + \cdots + \rho_\theta A_\theta, \\
B(p) &= B_0 + \rho_1 B_1 + \cdots + \rho_\theta B_\theta, \\
L_{cl}(p) &= \rho_1 b_{cl,1} + \rho_\delta b_{cl,\delta} + \rho_\theta b_{cl,\theta}, \\
L_T &= b_{\nu_T,0}.
\end{align*}
\] (27) (28) (29) (30)

The failure signature of elevator is parameter dependent since the direction of elevator depends on parameters. The direction of throttle failure does not depend on parameters. The $b_{cl}$ is that column of $B$ matrix, which represents the elevator actuator direction and $b_T$ is the column of $B$ matrix associated to throttle direction. The FDI filter is tested during an aircraft maneuver. The $\gamma$ command is a square wave, starting at 10 sec and ending at 40 sec. An 1 deg square wave elevator failure is simulated occurring from 15 sec to 45 sec and 10000 N step throttle failure occurring at 30 sec respectively. At present, the controller does not reconfigure after the elevator failure occurs. Thus the stabilizer is not used at any instance by the controller. The failures are modeled as an additive term in equation (26) corresponding to a loss in effectiveness of the control input channels. This means that the actuator effectiveness has been reduced to a constant value. In case of failures, the actuators are assumed to be able to work when faults have occurred. Although the actuator loses its effectiveness, the aircraft motion can be controlled with increased control action. The simulation results of the LPV FDI filter in case of closed-loop LPV
system can be seen in Figure 4.

![Graphs showing time and failures](image)

**Figure 4: FDI filter outputs using LPV model**

The first residual shows elevon fault and the second one is throttle fault respectively. The effect of the failures is decoupled and the residuals give an exact estimation of elevator and throttle fault. The impact of FPA command to residuals is negligible.

Next, the situation is studied when the FDI filter is applied to the nonlinear Boeing 747 simulation which represents the "true" system. Figure 5 shows the simulation results of the LPV FDI filter in case of closed loop nonlinear system.

The fault scenarios are same as in previous case. The effect of the failures is decoupled and the residuals give an exact estimation of failures. In case of nonlinear closed loop there is a bump at 10 sec and at 40 sec in elevator fault residual and a drift starting from 40 sec in stabilizer residual. The reason
of this fact is that the LPV FDI filter is analyzed using square wave $\gamma$ command. During climbing mode the trim conditions are changing that cause a transient alarm signal at residual outputs. However, when the $\gamma$ command goes back to zero indicating that the aircraft has reached its commanded altitude and the trim conditions do not change any longer, the fault residuals of FDI filter show only the effective faults. Unfortunately, if the trim conditions change in a long time duration, e.g. when we use a long $\gamma$ command for take off, then the transient false alarm signal appear for a long period. In other words, for the steady state trim conditions the LPV FDI filter works reliably. This fact can be seen at residual outputs after $\gamma$ command goes back to zero i.e. approximately at 60 sec. This means that the impact of FPA command in the nonlinear closed loop system appears in the residuals in a transient way and does not decrease the reliability of failure detection.
6 Conclusion

In this paper, a LPV FDI filter design based on extension of the fundamental problem of residual generation concepts elaborated for LTI systems has been presented through the application of LPV longitudinal model of Boeing 747-100/200. The LPV FDI filter used in this paper is sensitive for elevator and throttle failure. The LPV FDI filter is designed for an open loop LPV model of the system and then it is applied to the nonlinear closed loop system. In case of nonlinear closed loop, the impact of FPA command appears at residuals in a transient way, but this does not destroy the reliable operation of the LPV detection filter. For steady state trim conditions the LPV FDI filter works reliably.

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