Performance robustness analysis and control of aerospace vehicles: some feedback from the user point of view

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University of Bristol, 18-19 September 2017

Objective and outline

Objective: to present some application results on modeling, analysis and control of linear dynamic systems subject to parametric uncertainties or variations, with a focus on aerospace vehicle dynamics.

Outline

Robustness Analysis of Helicopter Ground Resonance with Parametric Uncertainties in Blade Properties

see also: L. Sanches, D. Alazard, G. Michon and A. Berlioz, *Robustness Analysis .... in Blade Properties*, Journal of Guidance, Control, and Dynamics, vol. 36 (n° 1).

Ground resonance: an unstable energy exchange between:

- rotor kinetic energy,
- body kinetic energy,
- potential energy stored in blade hinge stiffnesses and landing gear stiffness.

Illustration (credit Youtube!!)
see:
`https://www.youtube.com/watch?v=RihcJR0zvfM`

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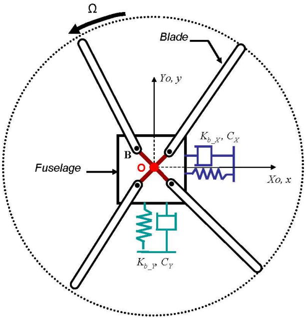
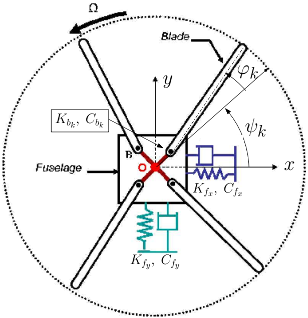


A simplified model.

Ground resonance dynamics model

LAGRANGE EQUATIONS: $\mathbf{M}(t)\,\ddot{\mathbf{q}} + \mathbf{G}(t)\,\dot{\mathbf{q}} + \mathbf{K}(t)\,\mathbf{q} = \mathbf{0}$
with: $\mathbf{q}\,(t) = [\ x(t) \quad y(t) \quad \varphi_1(t) \quad \varphi_2(t) \quad \varphi_3(t) \quad \varphi_4(t)\ ]^{\mathrm{T}}$.
State-space form: $\dot{\mathbf{x}} = \mathbf{A}_p(t)\mathbf{x}$ with $\mathbf{x} = [\mathbf{q}^{\mathrm{T}}\ \dot{\mathbf{q}}^{\mathrm{T}}]^{\mathrm{T}}$.
$\mathbf{M}$, $\mathbf{G}$, $\mathbf{K}$ and $\mathbf{A_p}$ are time-periodic: $\mathbf{A}_p(t+T) = \mathbf{A}_p(t)$ with $T = 2\pi/\Omega$.



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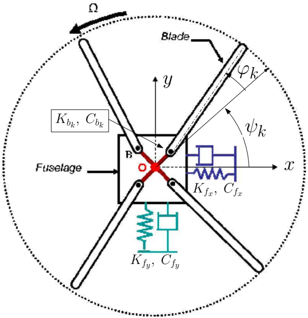
Stability by Coleman's approach:
if blade hinge properties are identical:
$C_{b_k} = C_b,\ K_{b_k} = K_b, \quad \forall\ k$

Then $\exists\ \mathbf{P}(t)$ s.t. $\mathbf{P}(t+T) = \mathbf{P}(t)$ and the mapping $\mathbf{q} = \mathbf{P}(t)\widetilde{\mathbf{q}}$ transforms the LTP model into a LTI model.
Then, stability analysis is obvious.



A simplified model.

Coleman, R., and Feingold, A.: Theory of Self-Excited Mechanical Oscillations of Helicopter Rotors with Hinged Blades, 1957.

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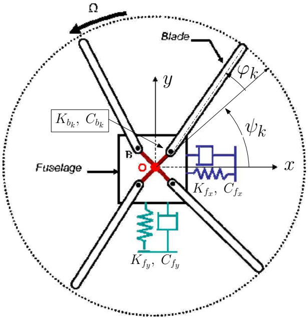
⇒ Does not work when hinge properties are not identical (due to aging effect)

⇒ **Floquet v.s.** μ**-analysis** of LTP system.



A simplified model.

Floquet analysis

Let us consider variations on each hinge stiffness: $K_{b_k} = K_{b_0}(1 + \delta_k)$,

Then: $\dot{\mathbf{x}}(t) = \mathbf{A}_p(t, \boldsymbol{\delta})\mathbf{x}(t)$ with: $\boldsymbol{\delta} = [\delta_1,\ \delta_2,\ \delta_3,\ \delta_4]^{\mathrm{T}}$. (1)

Transition matrix $\boldsymbol{\Phi}$: $\mathbf{x}(t) = \boldsymbol{\Phi}(t, t_0, \boldsymbol{\delta})\mathbf{x}(t_0)$.

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Then (1) is stable for a given $\boldsymbol{\delta}$ iff $\mathbf{R}(\boldsymbol{\delta})$ is Schur: $\equiv |\lambda_i(\mathbf{R}(\boldsymbol{\delta}))| < 1, \quad \forall\, i$

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$\mathbf{R}(\boldsymbol{\delta})$ can by approximated by $\mathbf{R}_{n_h}(\boldsymbol{\delta})$ using an oversampling period $h = T/n_h$:

$$\mathbf{R}_{n_h}(\boldsymbol{\delta}) = \prod_{i=0}^{n_h-1} \mathrm{e}^{\mathbf{A}_p(ih,\boldsymbol{\delta})h}\ .$$

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Parametric analysis: $\Rightarrow$ a gridding on $\boldsymbol{\delta}$ and a too high value on n_h ($n_h = 100$ for instance) is too CPU time-consuming.

Lifting procedure for μ-analysis

LFR of $\mathbf{A}_p(t,\boldsymbol{\delta})$: $\mathbf{A}_p(t,\boldsymbol{\delta}) = \mathbf{A}(t) + \mathbf{B}(t)\boldsymbol{\Delta}\mathbf{C}(t)$ with $\boldsymbol{\Delta} = \mathrm{diag}(\boldsymbol{\delta})$.
Let $\mathcal{M}(\mathrm{s},t) = \mathbf{C}(t)\left(\mathrm{s}\mathbf{1} - \mathbf{A}(t)\right)^{-1}\mathbf{B}(t)$, then:

$(\mathcal{M}(\mathrm{s},t), \boldsymbol{\Delta})$ is the LFR of the uncertain system.

Kim, J. *et Al.*, Robustness Analysis of Linear Periodic Time-Varying Systems Subject to Structured Uncertainty, SCL,2006

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- the n_h discrete-time LTI models are integrated over one period n_h $\Rightarrow$ $(\underline{\mathcal{M}}_d(\mathrm{z}),\underline{\boldsymbol{\Delta}})$ with $\underline{\boldsymbol{\Delta}} = \mathrm{diag}\left[\boldsymbol{\Delta},\ \boldsymbol{\Delta},\ \ldots,\ \boldsymbol{\Delta}\right]$,

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- re-ordering the inputs/ouputs: $\Rightarrow$ $(\widetilde{\underline{\mathcal{M}}}_d(\mathrm{z}),\widetilde{\underline{\boldsymbol{\Delta}}})$: the discrete-time lifted model with $\widetilde{\underline{\boldsymbol{\Delta}}} = \mathrm{diag}\left[\delta_1\mathbf{1}_{n_h},\ \ldots\ \delta_p\mathbf{1}_{n_h}\right]$,

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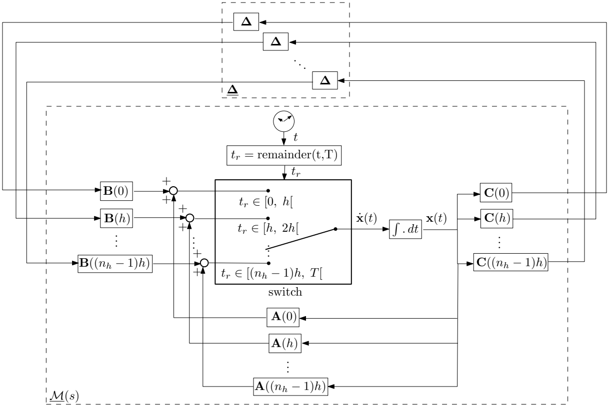
- inverse Tustin transformation is applied on $(\underline{\widetilde{\mathcal{M}}}_d(\mathrm{z},\underline{\widetilde{\boldsymbol{\Delta}}})$ to go back to continuous-time:

$\Rightarrow$ $(\underline{\widetilde{\mathcal{M}}}_c(\mathrm{s}),\underline{\widetilde{\boldsymbol{\Delta}}})$ is the **lifted** model.

Lifting procedure for μ-analysis

Up to the re-ordering on the augmented uncertainty block $\underline{\mathbf{\Delta}}$, the lifting procedure can be seen as the numerical integration of n_h-periodically switched LTI systems:



See also: `ltp2lti.m` **in** `https://personnel.isae-supaero.fr/daniel-alazard/matlab-packages/`.

Validation of the lifting procedure and discretization method

comparison

Considering: $\boldsymbol{\delta} = [0, 0, 0, \delta_4]$, $\delta_4 \in [-1 : 0.1 : 1]$:

- the discrete-time lifted model $\underline{\widetilde{\mathcal{M}}}_d(\mathrm{z})$ is computed for three different values of n_h $(10,\ 30$ and $100)$ and the three discretization methods (zoh, foh, Tustin),
- the LFT $\underline{\widetilde{\mathcal{M}}}_d(\mathrm{z}) - \underline{\widetilde{\boldsymbol{\Delta}}}$ is resolved and compared with the Floquet monodromy matrix $\mathbf{R}_{100}([0, 0, 0, \delta_4])$

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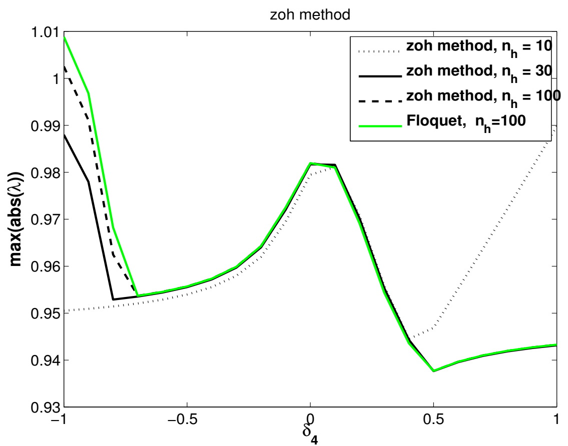
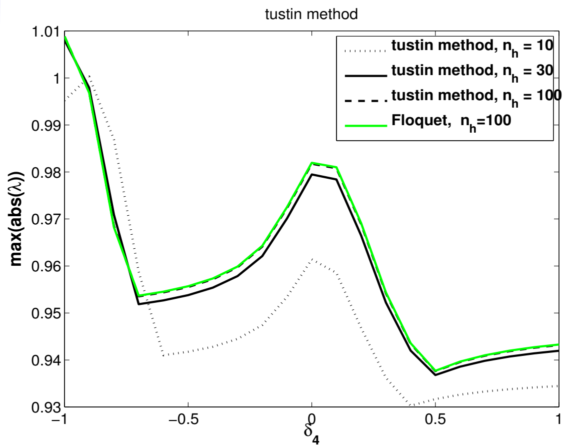
The comparison index is the highest eigenvalue (or characteristic multiplier) magnitude $\overline{|\lambda_l|}(\delta_4)$ and $\overline{|\lambda_{\mathbf{R}_{100}}|}(\delta_4)$ for the lifted model and the monodromy matrix, respectively.

Validation of the lifting procedure - ZOH methode



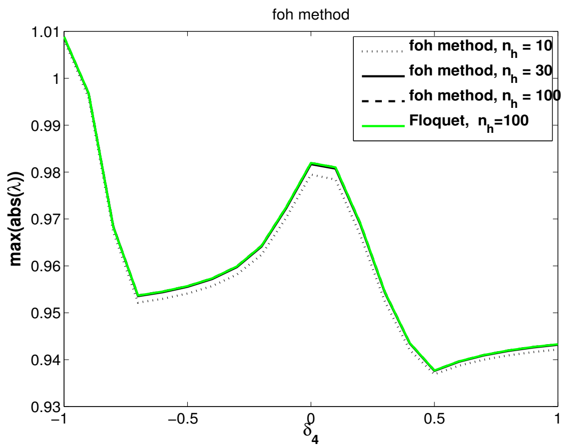
Evolution of the magnitude of the highest characteristic multiplier with respect to δ_4: $\overline{|\lambda_l|}(\delta_4)$, for different values of n_h using zoh method in the lifting procedure, and $\overline{|\lambda_{\mathbf{R_{100}}}|}(\delta_4)$.

Validation of the lifting procedure - Tustin method



Evolution of the magnitude of the highest characteristic multiplier with respect to δ_4: $\overline{|\lambda_l|}(\delta_4)$, for different values of n_h using tustin method in the lifting procedure, and $\overline{|\lambda_{\mathbf{R_{100}}}|}(\delta_4)$.

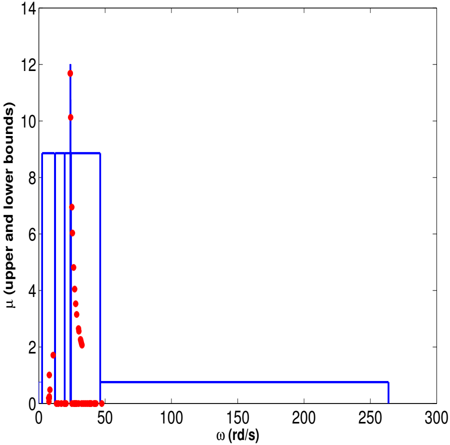
Validation of the lifting procedure - FOH method



Evolution of the magnitude of the highest characteristic multiplier with respect to δ_4: $\overline{|\lambda_l|}(\delta_4)$, for different values of n_h using foh method in the lifting procedure, and $\overline{|\lambda_{\mathbf{R_{100}}}|}(\delta_4)$.$\Rightarrow$ OK!! with $n_h = 30$.

μ-analysis results - Conclusions

μ-analysis is performed on the 12-th order $(\underline{\widetilde{\mathcal{M}}}_c(\mathrm{s}), \underline{\widetilde{\boldsymbol{\Delta}}}_{120\times 120})$ problem using the SMART toolbox.



$\boldsymbol{\delta}_{worst} = 0.085\,[1,\ 1,\ 1,\ 1]$: worst-case configuration corresponds to a rotor with identical hinge stiffnesses !!

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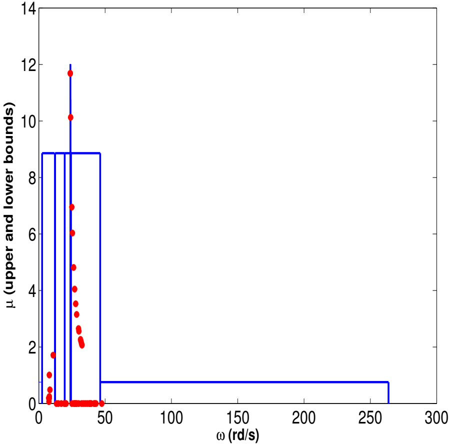

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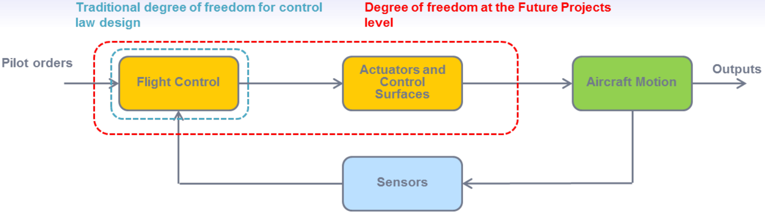
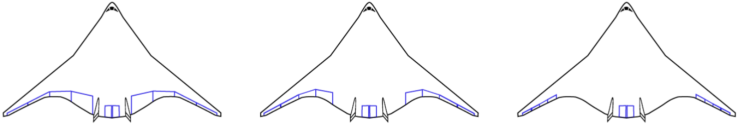
μ-analysis quite efficient and accurate but the Coleman method is still relevant!!

Outline

Preliminary design of control surfaces and laws



- Flying wing as a study case (strongly unstable and 3-axis coupled),
- Minimization of control surfaces size (η) under constraints of 3-axis control performance and max deflection (RMS) for given inputs (pilot orders and/or wind disturbance).

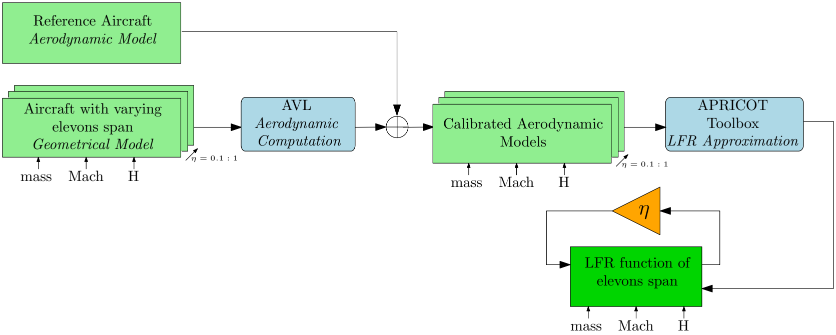


(a) $\eta = 1$ (b) $\eta = 0.8$ (c) $\eta = 0.4$

See also: Y. Denieul *et Al.*, *Multi-Control Surfaces Optimization for Blended Wing-Body under Handling Qualities Constraints*, Journal of Aircraft, 2017.

Preliminary design of control surfaces and laws

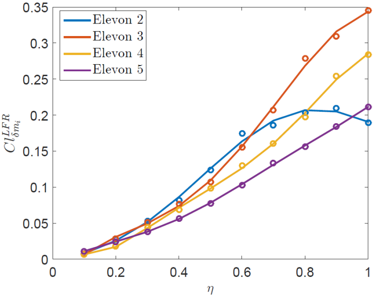
Computation of aerodynamic models for different control surfaces size η:



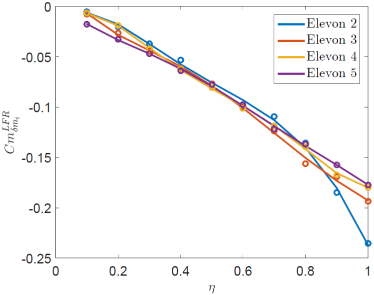
- APRICOT Toolbox used (Roos, Hardier, et Biannic 2014)
- Least-square extrapolation
- Final LFR size: order 20 with 5 order polynomial

Preliminary design of control surfaces and laws

Computation of aerodynamic models for different control surfaces size η,
LFR validation:

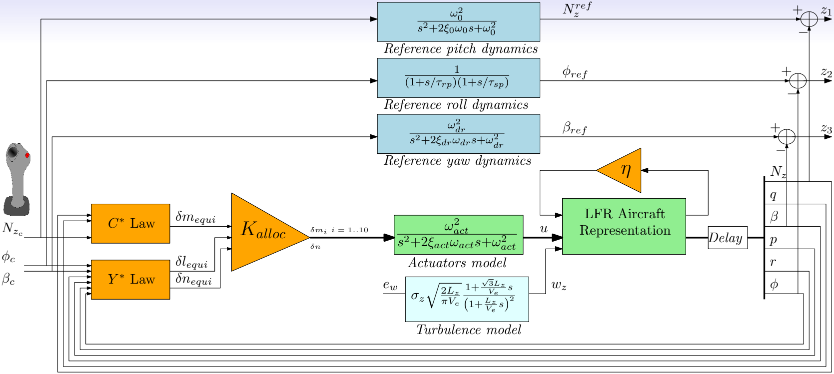


$Cm_{\delta m_i}$ approximation of elevon pitch gradient for varying η.



$Cl_{\delta m_i}$ approximation of elevon roll gradient for varying η.

Elevon size η and 3 axis control law co-design



Structured control laws: $[C^\star\text{Law}, Y^\star\text{Law}] = fct(\mathbf{K})$. Then for a given γ:

Co-design for handling qualities:

Solved using SYSTUNE routine from Matlab RCT (Apkarian et Noll 2015)

$$(\widehat{\eta}, \widehat{\mathbf{K}}, \widehat{K}_{alloc}) = \arg \min_{\eta, \mathbf{K}, K_{alloc}} \eta \; / \; \|T_{(Nz_c, \phi_c, \beta_c) \to (z_1, z_2, z_3)}\|_\infty < \gamma \, .$$

Co-design with all flying qualities and constraint

	Function / Variable	Description	Quantity
minimize	η	Outer elevons total span	
with respect to	K	Control law gains	16
	K_{alloc}	Control allocation matrix	11
	η	Outer elevons total span	1
subject to	$\lVert \frac{1}{\Delta\delta m_i^{max}} T_{Nz_c \to u} \Delta\alpha \frac{V_e}{g} z_\alpha \rVert_\infty \leq 1$	Maximum deflection in response to longitudinal order.	5
	$\lVert \frac{1}{\dot{\delta m}_i^{max}} T_{Nz_c \to \dot{u}} \Delta\alpha \frac{V_e}{g} z_\alpha \rVert_\infty \leq 1$	Maximum deflection rate in response to longitudinal order.	5
	$\lVert \frac{2}{\Delta\delta m_i^{max}} T_{e_w \to u} \rVert_\infty \leq 1$	Maximum deflection in response to longitudinal turbulence	5
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	$\lVert \frac{1}{\Delta\delta m_i^{max}} T_{\phi_c \to u} \phi^{max} \rVert_\infty \leq 1$	Maximum deflection in response to bank order.	5
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	$\lVert T_{(Nz_c, \phi_c, \beta_c) \to (z_1, z_2, z_3)} \rVert_\infty \leq \gamma$	Optimal closed-loop performance.	1
	$\forall p, p$ pole of $P(s)$: $Re(p) \leq -$MinDecay $Re(p) \leq -$MinDamping.$\lvert p \rvert$	Closed-loop poles location.	1
	K internally stabilizes $P(\eta)$		

$$\widehat{\eta} = 0.3885 \; .$$

Thank you !

Questions ?