Frozen-time and time-varying robust stability certificates for the coupled pitch/yaw motion of an axial-symmetric launch vehicle

Tiziano Pulecchi*, Andrés Marcos*,
Christophe Roux, Valerio Mantini**, Samir Bennani*****

**DEIMOS Space S.L.U., 28760 Tres Cantos (Madrid), Spain*
e-mail: tiziano.pulecchi@deimos-space.com,
***ELV, Corso Garibaldi 22, 00034, Colleferro (Rome), Italy*
**** ESA/ESTEC, Keplerlaan 1, 2201 AZ, Noordwijk, The Netherlands.*

Abstract: Clearance of Thrust Vector Control laws for launchers demands the capability to provide analytical certificates for the system's robust stability with respect to nonzero roll rates. The current Verification and Validation approach used in industry is limited in as much as it entails evaluation of gain margins for a finite set of conditions and assuming a frozen-time setting. Advanced analysis techniques, such as mu analysis and Integral Quadratic Constraints, have the potential to clear the whole analysis space and take into consideration nonlinear and time-varying aspects. The contribution of this paper stands in exploiting such potential to provide frozen-time and time-varying robust stability certificates for the coupled pitch/yaw attitude motion of an axial-symmetric launch vehicle.

1. INTRODUCTION

Clearance of Thrust Vector Control (TVC) laws for launchers demands the capability to provide analytical certificates for the system's robust stability with respect to nonzero roll rates. This issue takes on special relevance in that the TVC control laws of an axial-symmetric launch vehicle (LV) are typically designed in a SISO setting under the assumption of perfect pitch/yaw motion decoupling.

The Verification and Validation (V&V) approach currently undertaken in industry entails evaluation of gain margins for a finite set of roll rate values. Robustness of the coupled pitch/yaw system is verified opening one loop-at-a-time on a finite set of sample points. Although flight-proven in Europe and USA, this approach is nevertheless quite restrictive.

Advanced analysis techniques such as μ analysis (Doyle 1982, Packard 1988, Packard 1990, Balas 1992, Beck 1992) and Integral Quadratic Constraints (IQC) (Megrestki 1997, Jönsson 2001, Kao 2007, Koroglu 2006), on the other hand, allow clearance of the complete analysis space in a single attempt. Specifically, for the LV problem considered, μ analysis has the potential to detect the maximum roll rate (PMAXGRS) for which the system is guaranteed to remain stable, the minimum guaranteed destabilizing roll rate (PMINGD) and single and multi-loop gain and delay margins. Furthermore, such frozen-time robust stability certificates can be extended to a time varying setting using IQC analysis. As a result, robust stability can be proved for a compact region in the roll rate/roll acceleration phase plane.

In this article, the aforementioned modern control analysis techniques are used to provide robust stability certificates for the coupled pitch/yaw motion of an axial-symmetric launch vehicle. Although this benchmark problem is defined based on the VEGA launcher (Roux07), its characteristics are representative of a generic axial-symmetric launch vehicle.

The outline of this paper is as follows. In Section 2 an overview of the problem is given. Section 3 presents the launcher model used for the analysis, which is based on linear fractional transformations (LFT). Section 4 presents the analysis results starting with mu analysis results and followed by the IQC analysis. Finally, conclusions are drawn and future developments are discussed in Section 5.

2. LAUNCH VEHICLE ATTITUDE DYNAMICS

2.1 The open loop plant

TVC control is used to track a reference (optimal) trajectory in space by suitably controlling the orientation of the rocket engine's nozzle relative to the launch vehicle itself. For low roll rates and because of the vehicle axial symmetry, the small-amplitude pitch, yaw, and roll motions can be regarded as being virtually decoupled. This is the reason why, for this class of launch vehicles, the TVC control laws applicable to the pitch and yaw axis are identical and their design is based on a decoupled model. Even so, during the Design Verification and Validation (DV&V) phase the fully roll-coupled pitch/yaw dynamics must be addressed.

The LV coupled pitch/yaw rigid body motion dynamics is completely described by its pitch and yaw attitude (θ and ψ) and linear motion (y and z) in a frame linked to the velocity of the reference trajectory.

Under the assumption of a perfect axial-symmetric launch vehicle, the dynamic equations for the pitch and yaw dynamics, expressed in the Linear Parameter Varying (LPV) formalism are the following:

$$\begin{aligned} \dot{x} &= A(p)x + Bu \\ y &= Cx \end{aligned} \tag{1}$$

$$x = [\, r \quad q \quad \psi \quad \vartheta \quad \dot{z} \quad \dot{y} \quad z \quad y \,]$$
$$u = [\, \beta_\psi \quad \beta_\vartheta \,] \tag{2}$$
$$y = [\, \psi \quad \dot{z} \quad z \quad \vartheta \quad -\dot{y} \quad -y \,]$$

$$A = \begin{bmatrix} 0 & \lambda p & A_6 & 0 & A_6/V & 0 & 0 & 0 \\ -\lambda p & 0 & 0 & A_6 & 0 & -A_6/V & 0 & 0 \\ 1 & 0 & 0 & p & 0 & 0 & 0 & 0 \\ 0 & 1 & -p & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -a-\gamma & 0 & a/V & -p & 0 & 0 \\ 0 & 0 & 0 & a+\gamma & p & -a/V & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \tag{3}$$

$$B = \begin{bmatrix} k_1 & 0 \\ 0 & k_1 \\ 0 & 0 \\ 0 & 0 \\ -\gamma_t & 0 \\ 0 & \gamma_t \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}$$

The system matrices [A,B,C,D] depend on a set of variables, which can be regarded as known constants once a linearization condition is selected. These parameters include: the launch vehicle's velocity V, inertia ratio factor $\lambda=1-I_{xx}/I_{yy}$, aerodynamic instability coefficient A_6, lift force coefficient a, accelerations γ and γ_t, and controllability coefficient k_1. Numerical data for their computation were provided by ELV for the linearization condition selected for validation of the proposed approach (P80 phase). Notice that, once the desired linearization point is selected, the open-loop plant depends parametrically only on the roll rate p. Also notice that, although we limit our exposition to a single flight point, a batch of conditions should be analyzed to properly account for the time-varying aspects of LV dynamics.

As shown in equation (2), the system state gathers pitch q and yaw r rates, yaw ψ and pitch θ, and the launch vehicle's linear motion in the z and y directions. The system inputs are the commanded TVC deflections in the pitch β_θ and yaw β_ψ directions.

The open-loop plant is further complemented by actuators and sensor dynamics (modeled by a four-states/two-zeroes SISO transfer function) and an on-board computer (OBC) transmission delay (modeled by a non-minimum phase lead-lag filter with time constant τ). Identical actuator and sensor models, as well as transmission delays, are assumed for the pitch and yaw channels.

2.2 TVC controller

The digital TVC controller commands a deflection in pitch and yaw of the thrust vector. The same control law is used for each channel. Such a control law gets as input the relevant attitude angle, position and velocity information and provides as output the desired nozzle deflection. Prior to analysis, the digital controller is converted into continuous form using a Tustin transformation. The high-frequency mismatch introduced by the conversion of the digital controller into continuous form does constitute here a minor limitation, as the present analysis focuses on an early phase of flight (the first of three solid rocket stages), characterized by strong aerodynamic forces acting on a relatively rigid system.

3. LINEAR FRACTIONAL REPRESENTATION

Modern analyses techniques such as mu and IQC analysis builds upon the Linear Fractional Representation (LFR) paradigm (Packard 1988, Balas 1992). In a nutshell, a LFR is the representation of a feedback system that can be described as the feedback interconnection of two causal and bounded mathematical operators: a feed-forward operator M and a feedback operator Δ used to describe the model uncertainties, nonlinearities and time variability. Within this framework, systems are regarded as operators mapping one normed vector space into another, whereas the input/output signals belong to these vector spaces.

Two LFR models are used for the analyses presented in Section 4. The first model depends on a single parameter, the roll rate p, and is used for the robust stability analyses presented in sections 4.1 and 4.3. The second LFR model is used for gain margin evaluations (Section 4.2) and augments the first model with two additional parameters: the artificial loop gains G_1 and G_2 on the pitch and yaw control loops. A symmetric range is adopted for the roll rate $p \in [-40,+40]$ degrees per second, with the nominal value set to zero. The artificial loop gains G_1 and G_2 are assumed to take nominally a value of 1 and vary in the range $[0,2]$ or ±100% with respect to their nominal value. Note that this choice is meaningful in a robust analysis framework in that $G_1=2$ (respectively, $G_2=2$) implies a 6dB gain margin for the pitch (respectively, yaw) control loop, a value typically sought after in control system specifications.

4. ANALYSIS RESULTS

4.1 Robust stability results

The μ analysis toolbox (Balas 1992) is used to assess the maximum roll rate level (PMAXGRS) for which the fully coupled MIMO system is guaranteed to be stable (in a frozen time setting) and the minimum roll rate level (PMINGD) for which the system is known to be destabilized.

A μ peak lower than unity (Fig. 1) implies that the system is robustly stable (in an LTI sense) to all roll rates inside the admissible range. Furthermore, it also guarantees the system to be stable with respect to all perturbations of size PMAXGRS = $1/\mu_{peak}40$ = 70.1 degrees per second. By contrast, a traditional LTI analysis yields that the exact roll rate value above which the system becomes unstable is around 97.5 degrees per second (Fig. 2). This mismatch is indicative of the conservative nature of μ.

The blue line (Fig. 2) represents the system's spectral abscissa (maximum of system eigenvalues' real part) versus assigned roll rate value, as computed by the array of LTI analyses. Note how the system's spectral abscissa quickly approaches zero (corresponding to a migration of the less stable pole towards the imaginary axis) already for low values of the system roll rates, but stays close to the instability bound as if in a plateau until a sudden instability departure beyond roll rates of 97.5 degrees per second.

Results from mu analysis are superimposed on the very same plot. The green area, corresponding to roll rates lower than 70.1 degrees per second, represents the interval of roll rates cleared by mu (upper bound) computation. Notice that the close-to-zero plateau results in this case in a cleared area smaller than the real (in a LTI sense) one. Because of numerical difficulties in computing an accurate lower bound, PMINGD could not be assessed.

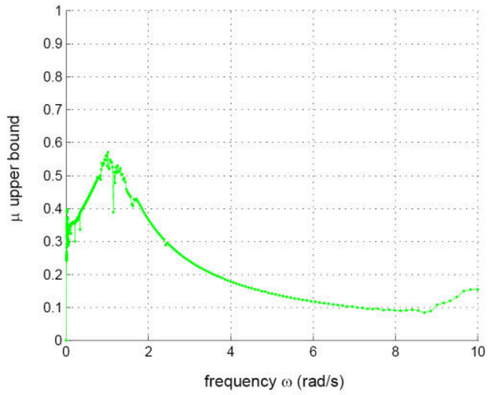

Fig. 1. Mu upper bound vs. frequency.

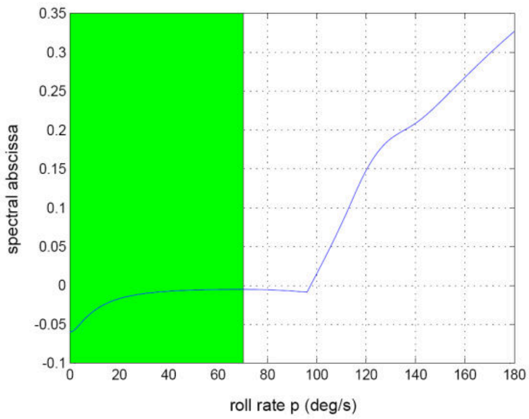

Fig. 2. Stability degree vs. positive roll rate (blue line). Mu results are given by the shadowed region (in green).

4.2 Gain margin analysis

Single loop at a time and/or multi-loop gain margins can be effortlessly derived via mu analysis by introducing a fictitious (uncertain) loop gain in the analysis model (Fig 3).

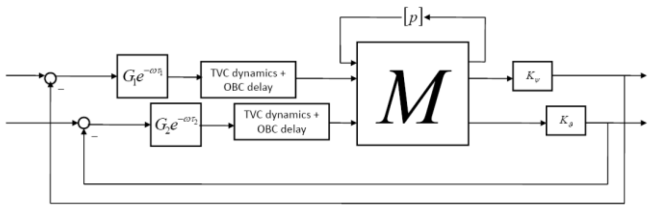

Fig. 3. MIMO closed-loop LFT with TVC dynamics, OBC delays, and artificial gains G_* and time delays τ_*.

Gain margin (GM) is a concept traditionally used in a single loop at a time framework. By contrast, in a multi loop setting several possibilities are available. In compliance with the choice made in (Roux 2007), a diagonal gain matrix is adopted for the present analyses.

It is noted that the system's GM depends on the level of roll rate the coupled pitch/yaw dynamics is required to withstand. Because roll rate couplings act as a destabilizing term, the higher the roll rate, the smaller the GM is in general. In the context of this work, multi-loop GM are evaluated for several levels of roll rate *(|p|<10, |p|<20, |p|<30, |p|<40* degrees per second). For all such cases, the analysis succeeds in evidencing large enough margins always exceeding 2.8 dB, or ±38% loop gain, in the MIMO case (Fig. 4).

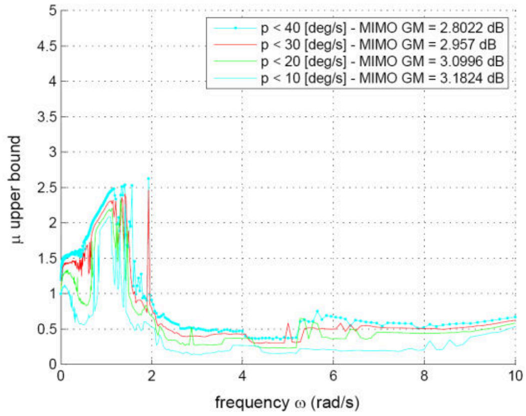

Fig. 4. Multi loop GM and associated roll rate levels.

Notice how the strongest gain margin result is obtained for the smallest level of roll rate considered. Simply stated, that is to say that reducing the roll rate allows for larger gain margins.

Gain margins computed via mu analysis can be graphically confirmed on a Nichols chart. The LPV system, eqs (1-3), is evaluated on a finite set of roll rates, and the corresponding transfer functions plotted on the Nichols chart. The output of the analysis is depicted in Fig. 5, where the red area (only the top and bottom lines are meaningful for GM purposes) represents the minimum guaranteed gain margin as provided by mu analysis (±2.8 dB). As you can see, no transfer function ever intersects the red region identified by mu for the tested roll rates.

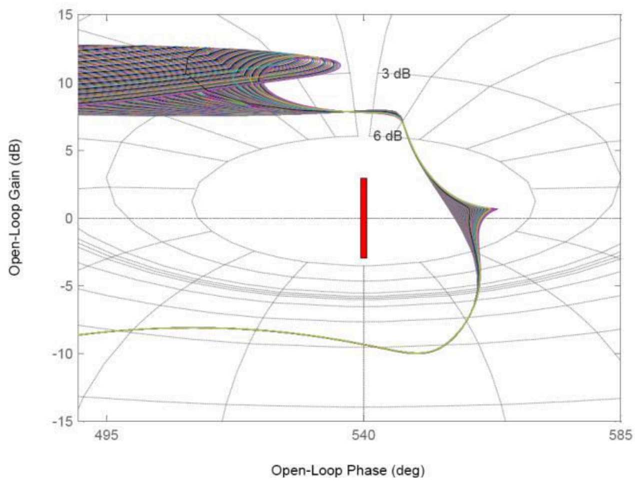

Fig. 5. Nichols chart corresponding to roll rates $|p|\leq15$ degrees per second. The red area represents the minimum GM guaranteed by mu analysis.

Note that gain margin results provided by mu analysis are also quite conservative when compared to the indications that can be extracted from reading the Nichols chart itself. Indeed, when $|p|<15$ degrees per second, inspection of the Nichols chart in Fig. 5 yields a gain margin of 7.77 dB, which more than doubles the mu analysis result (2.8 dB) corresponding to the same range of $|p|$. Nevertheless, this interpretation may well be misleading since the Nichols results pertain to SISO with one loop-at-a-time analysis while the mu-analysis results apply to the MIMO system –the former are reported only for comparison purposes.

4.3 Time-varying stability analysis

IQC theory can be used to extend mu analysis robust stability results and determine up to what level of roll rate and roll acceleration the system of Eq (1-3) is robustly stable. Indeed, IQC allows analyzing robust stability in a time-varying setting. In this context, time variability is embedded in the system's parameter, whose rate of variation is bounded.

In the IQC framework, the robust stability analysis problem is converted into an IQC feasibility test. The IQC feasibility test is in turn recast as an LMI optimization problem with decision variable the Lyapunov matrix P (of dimension n^2, with n = number of overall system's states). If the outcome of the test is positive, the system's robust stability is proved. If, on the other hand, it is negative, the test is inconclusive and nothing can be said about the system's stability.

Two classes of IQC analyses were performed using the LPVMAD-IQC toolbox developed by Prof. Carsten Scherer [Veenman09]. First, the roll rate is assumed to be a (static) uncertain parameter of known maximum value. These results can be directly mapped to the mu robust stability results. Second, the roll rate is assumed to be a time varying parameter with assigned bounded rate of variation.

Because of its trial-and-error nature, the procedure adopted in selecting the analysis conditions for the IQC feasibility test was in part heuristic. Roll rate bounds of 15, 30, 45, 60, 75, 90 and 105 degrees per second and roll acceleration bounds of 0 and 5 degrees per second square are initially considered for IQC analysis. For all such conditions an IQC test is run. Based on the outcome of the IQC test, new conditions are generated and analyzed for different values of maximum roll acceleration bounds (and possibly on a reduced number of maximum roll rate bounds). Proceeding in this fashion, the computational effort is minimized by populating iteratively the grid of analysis conditions based on the outcome of the previous IQC tests.

Notice that, while mu analysis can clear in one test whole regions of the parameter space (of course restricted to a frozen time interpretation of the space), IQC analysis can only provide stability results on a pointwise basis. This means that failure of the IQC feasibility test does not imply instability of the analyzed condition. Instead, it simply implies that the algorithm failed to converge to a solution of the optimization problem.

Twenty-six IQC tests were run in order to clear the yellow area in Fig. 6 (IQC test successful). Green circles (respectively, red crosses) correspond to points in the roll rate/roll acceleration plane where the IQC feasibility test was positive (respectively, negative). With this respect, note how the frozen-time robust stability results provided by mu analysis (green line at the bottom axis in Fig. 6) can be dramatically enhanced by use of IQC analysis (yellow region in Fig. 6).

Thus, not only does IQC analysis allow tackling the more complex time varying parameter case, it also improves over the mu analysis (frozen-time setting) results by guaranteeing the system robust stability with respect to all roll rates of amplitude less than 95 degrees per second. This is quite an improvement over the 70.1 degrees per second value guaranteed by mu in Section 4.1 and indeed very close to the 97.5 degrees per second of the traditional LTI analysis. The price to be paid for this reduced conservatism stands in the iterative nature of the approach itself.

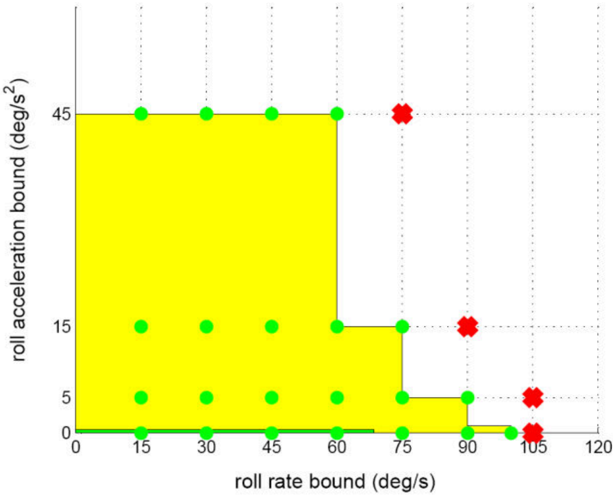

Fig. 6. IQC robust stability region (yellow) vs. mu analysis results (green line @ zero roll acceleration).

5. CONCLUSIONS

The work presented is part of a European Space Agency's study entitled "Robust Flight Control System Design Verification and Validation" (RFCSDVV). Its objective is to develop an Enhanced Design Validation and Verification Framework, including methodology, algorithms and tools for uncertain, nonlinear and time-varying safety critical systems such as Launcher Vehicles. Under these auspices, the current work intends to extend (by means of advanced analysis techniques such as mu and IQC analyses) the frozen-time approach currently used to quantify the effect of roll coupling on the pitch/yaw stability margins to the bounded uncertain/time varying parameter cases. The results obtained were contrasted against standard analysis results and show great promise to be incorporated into the current V&V process.

In summary, μ analysis has been used to verify up to what level of roll rate the system can be guaranteed to remain stable. Subsequently, an uncertain gain was introduced and μ analysis was used once again to deduce the model's gain margins. Finally, IQC analysis was exploited to demonstrate the system's robust stability in a time varying setting. That is, a whole region in the roll rate/roll acceleration phase plane was cleared for flight with the provided TVC control law.

The analyses were performed under the assumption that all model parameters, but the launch vehicle's roll rate, were exactly known. Assuming that the model coefficients be perfectly known is limiting. For example, the aerodynamic instability coefficient A_6 depends on a number of uncertain factors, including the position of the launch vehicle's center of pressure and center of gravity, dynamic pressure and normal force coefficient slope. On the other hand, precise determination of the controllability coefficient k_1 is affected by propulsive scatterings and uncertainty on the launch vehicle's center of gravity. Extension of the analysis results provided in this paper to the uncertain A_6, k_1 case is currently the subject of an ongoing investigation.

ACKNOWLEDGEMENTS

This work is funded by the European Space Agency under ESA-ESTEC contract AO-1-6322/09/NL/JK.

The first two authors would like to acknowledge the use of Prof. Carsten Scherer's LPVMAD-IQC toolbox.

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