WORST-CASE ANALYSIS OF FLIGHT CONTROL LAWS FOR RE-ENTRY VEHICLES

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Abstract: This paper reports results of a joint study between ESA, DEIMOS Space S.L. and the University of Leicester on the robustness analysis of flight control laws for future hypersonic re-entry vehicles. We apply a novel hybridised version of the deterministic global optimisation algorithm, DIviding RECTangles (DIRECT), to perform worst-case analysis of a nonlinear-dynamic inversion (NDI) flight control law for a highly-detailed simulation model of a hypersonic re-entry vehicle. Nonlinear clearance criteria widely used in the European aerospace industry for the clearance of flight control laws for highly manoeuvrable aircraft are developed and applied in the context of the re-entry vehicle flight control problem. The proposed approach is shown to have the potential to improve significantly both the reliability and efficiency of the flight clearance process for future re-entry vehicles. *Copyright ACA 2007*

Keywords: Hybrid Optimisation, Robustness Analysis, Simulation, Re-entry Vehicle

1. INTRODUCTION

Atmospheric re-entry is an important and critical part of the Reusable Launch Vehicle (RLV) mission. During the re-entry flight phase, the space vehicle follows a predefined trajectory towards the designated landing point, travelling from space to the dense atmosphere of earth. As a result, the vehicle is subjected to high levels of uncertainty and variations in key flight parameters during the course of its mission. A primary requirement for re-entry guidance and flight control laws is that they exhibit sufficient levels of robustness to allow close tracking of the pre-defined trajectory in spite of high levels of uncertainty and disturbances. In order to demonstrate that this requirement is satisfied, maximum deviations from the prescribed trajectory due to uncertainty in flight parameters such as mass, centre-of-gravity locations, inertias and aerothermodynamic parameters, as well as actuator and sensor uncertainties need to be precisely evaluated in simulation, prior to any test flight. This process of "flight clearance must be carried out in all normal and various failure conditions, and in the presence of all possible parameter variations.

The task of analysing and quantifying the robustness properties of the RLV flight control algorithms is a very lengthy and expensive one, where different combinations of large numbers of uncertain parameters must be investigated such that an estimate about the worst case stability and performance of the control laws can be made. For nonlinear flight clearance problems, the current industrial standard is to use a gridding approach, where either the clearance criteria are evaluated for all combinations of the extreme points of the vehicle's uncertain parameters or Monte-Carlo simulation is employed to randomly sample the parameter space, (Fielding *et al.*, 2002). Unfortunately, the computational effort involved in the resulting clearance assessment increases exponentially with the number of uncertain parameters that are to be considered (combinations of extreme points) or with the desired confidence levels for the clearance results (Monte-Carlo simulation). Another difficulty with these approaches is the fact that there is no guarantee that the worst case uncertainty combination has in fact been found, since it is possible that the worst-case combination of uncertain parameters does not lie on the extreme points, or in the sampled set used by Monte-Carlo approaches. A promising approach to address the above difficulties is to use advanced optimisation algorithms to search the parameter space for worst-cases that violate the particular clearance criterion under investigation. Clearly, given that the parameter space for this type of problem will in general be highly nonlinear and non-convex, (Forssell, 2003), global optimisation methods will be required to avoid getting trapped in locally optimal solutions. Previous work by the authors has explored the applicability of various evolutionary optimisation methods to the flight clearance problem, and has shown that, when hybridised with appropriate gradient-based algorithms, they have the potential to improve significantly both the reliability and efficiency of the flight clearance process, (Menon *et al.*, 2006*a*; Menon *et al.*, 2006*b*).

In this paper, the flight clearance problem for a highly detailed simulation model of a generic RLV over a lower atmospheric phase of its re-entry trajectory is considered. The flight control law included in the model has been designed using nonlinear dynamic inversion (NDI) methods to provide robust trajectory tracking over the specified flight phase. The clearance problem is solved using a novel hybridised version of the deterministic global optimisation scheme known as as DIviding RECTangles (H-DIRECT). The DIRECT algorithm is based on Lipschitzian optimisation, which unlike evolutionary global optimisation methods comes with deterministic proofs of convergence to the global solution. To reduce the computational complexity of the approach, the DIRECT algorithm is hybridised with a local optimisation method known as Pattern-Search. The resulting hybrid algorithm appears to be particularly well suited for application to flight clearance problems, due to its deterministic nature, proven convergence guarantees and computational efficiency.

2. RLV MODEL, CONTROL LAW AND CLEARANCE CRITERION

The generic RLV high-fidelity simulation model is based on the HL-20 aerodynamic database and X38-type geometric and aerodynamic surface configuration, and has a dry mass of 19,100-lb. This simulation model has been developed by DEIMOS Space S.L. for the European Space Agency (ESA) to act as a research platform for the investigation of re-entry and autoland guidance, navigation and control systems, (Fernandez *et al.*, 2006).

The model consists of a reference trajectory generator, a nonlinear dynamic inversion (NDI)-based flight

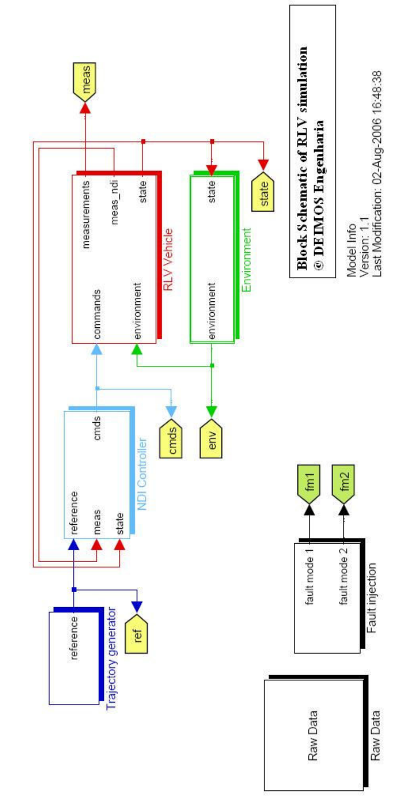

Fig. 1. Block schematic of RLV

control system, nonlinear actuator models, the RLV dynamics, sensors such as gyros and accelerometers, and detailed environment models (US standard 1969 and Earth gravity and geoid models). Figure 1 shows a block diagram schematic of the RLV simulation model, which is implemented in the Matlab Simulink environment. The reference trajectory is defined in terms of Angle of Attack (AoA or α), Angle of Side Slip (AoSS or β), and bank angle ϕ. The NDI controller provides the elevator, aileron, rudder and brake control inputs according to the desired dynamics. The controller also includes actuator allocation functions depending on the commanded moments, altitude and velocity of the RLV. More details of the model and its associated flight control system are available in (Fernandez *et al.*, 2006). The parameters in the model, and associated uncertainty values, are accessible through a database consisting of a collection of XML files accessible by the user.

The complete re-entry trajectory for the vehicle takes 1680 seconds of simulation time and is divided in flight phases based on dynamic pressure and atmospheric layer. The present analysis focuses on a lower atmosphere phase starting at 1588 seconds and ending at 1675 seconds that covers the 32 to 20 km altitude range. The reference trajectory in this segment includes a reduction of AoA from 30 degrees to nearly 20 degrees, while keeping a zero AoSS and with a defined bank angle variation. The description and allowed ranges of the uncertain parameters considered for the present analysis are given in Table 1. As can

Table 1. RLV Model Uncertain Parameters

Parameter	Bound	Description
Δ_{mass}	[-2313.3, 2313.3]	variation in RLV dry mass from nominal one (11566.55 kg)
Δ_{Ixx}	[-1627, +1627]	variation in Moment of inertia about X (8135.0 $4kgm^2$)
Δ_{Iyy}	[-15185, +15185]	variation in Moment of inertia about Y (75926.0 kgm^2)
Δ_{Izz}	[-15863, +15863]	variation in Moment of inertia about Z (79315.0 kgm^2)
Δ_{Ixz}	[-628.8, +628.8]	variation in Product of inertia XZ (3144.0 kgm^2
$\Delta_{x_{cog}}$	[-0.4912, +0.4912]	variation in X centre of gravity from nominal one (4.9213 m)
$\Delta_{y_{cog}}$	[-0.01, +0.01]	variation in Y centre of gravity from nominal one (0.0 m)
$\Delta_{z_{cog}}$	[-0.1009, +0.1009]	variation in Z centre of gravity from nominal one (1.0094976 m)

be seen from the table, in the present analysis we have focused mainly on uncertainty in the parameters representing the vehicle's mass, inertias and centre-of-gravity. For the final version of the paper, our analysis will include many other uncertain parameters, including pitch, roll, yaw and sideforce stability derivatives, sensor errors and noise, atmospheric disturbances and several fault/failure cases.

To analyse the robustness of the NDI control law in tracking AoA trajectories over the considered flight phase, a cost J is defined by equation(1),

$$J = \|\alpha_{ref} - \alpha_{\Delta}\|_{\infty} \quad (1)$$

where α_{ref} represents the reference AoA trajectory and α_{Δ} represents the actual AoA trajectory followed by the vehicle in simulation. This particular clearance criterion was chosen initially for this study as criteria of this type are widely used throughout the European aerospace industry for the clearance of flight control laws, (Fielding *et al.*, 2002; Forssell, 2003). The uncertain parameter vector Δ consist of the parameters defined in Table 1, and its dimension is hence fixed at 8. The worst-case analysis problem is posed as identifying the Δ^* vector such that the following maximisation problem is solved.

$$\max J = \|\alpha_{ref} - \alpha_{\Delta}\|_{\infty} \quad (2)$$

$$\text{sub. to } \underline{\Delta} \leq \Delta \leq \overline{\Delta} \quad (3)$$

where $\underline{\Delta}$ and $\overline{\Delta}$ define the lower and upper bounds on the uncertain parameters. The Δ^* producing the maximum cost value $J*$ corresponds to the uncertain parameters that give the maximum deviation from the reference trajectory α_{ref}. The resulting optimisation problem is obviously highly nonlinear and nonconvex in general. Note that in this preliminary version of the paper we focus on a clearance criterion involving AoA only. In the final version of the paper we will include analysis results for a number of different clearance criteria, in order to perform a more complete evaluation of the robustness of the NDI flight control law.

3. DETERMINISTIC GLOBAL OPTIMISATION

To solve the optimisation problem defined in the previous section, we use the optimisation algorithm DIviding RECTangles (DIRECT), which is a deterministic global optimisation algorithm developed by (D.R.Jones *et al.*, 1993). DIRECT is a modification of the classical one-dimensional lipschitzian optimisation algorithm known as the Shubert algorithm (Shubert, 1972; Pinter, 1986). To date, the DIRECT algorithm has been successfully applied to several different classes of engineering problems. In (H.Zhu and D.B.Bogy, 2002), DIRECT optimisation was applied to a slider air-bearing surface (ABS) design, an important problem in magnetic hard disk drives where the cost function evaluation requires a substantial amount of computation time. Fast convergence of the algorithm and a favourable comparison with an adaptive simulated annealing algorithm were demonstrated in that study. In (Carter *et al.*, 2001), the problem of minimising the cost of fuel and/or electric power for the compressor stations in a gas pipeline network was addressed using the DIRECT algorithm and a hybrid version of DIRECT with implicit filtering. Again, in that study, promising results were obtained, with the hybridisation of the basic global algorithm significantly improving the convergence to the global solution in the presence of noise. In (Zwolak *et al.*, 2005) the DIRECT algorithm was applied to the problem of parameter identification in a nonlinear cell cycle model, i.e. to search for the globally optimal kinetic rate constants for a proposed mathematical model of a biochemical network.

The DIRECT optimisation/search method does not require any derivative information to be supplied, and uses a centre point sampling strategy. Furthermore, it assumes that every variable is in the closed set $[0,1]$ without loss of generality (in our formulation normalisation of the variables to this interval can always be done). Thus, the search space is an n-dimensional hypercube or box, defined as $D = \{x \in \Re^n : 0 \leq x_i \leq 1\}$. The algorithm works in the normalised parametric space, transforming to the actual search space as and when the cost function is to be evaluated. The main idea of the algorithm is as follows. As the algorithm proceeds, the search space is partitioned into smaller hypercubes or boxes and each is then sampled at the centerpoint of the interval. Over iterations, the algorithm tries to find all the potentially optimal hypercubes or boxes in the search space and then partition them, thereby obtaining the global solution. The main stages of the algorithm are now described in detail.

3.1 Centre point sampling and dividing strategy

The algorithm begins with the evaluation of the cost function about the centre point, say c, of the normalised search space. The subsequent step is to divide the hyper box. We sample the points $c \pm \delta e_i$,

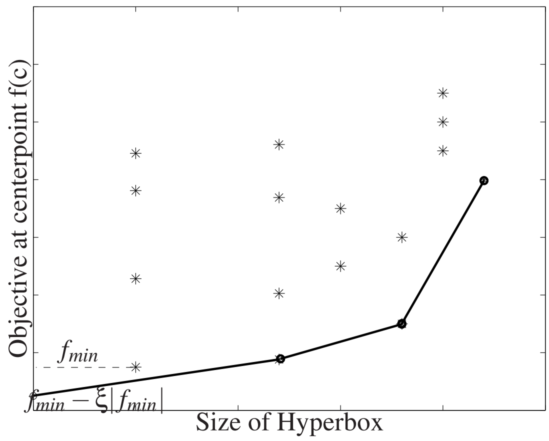

Fig. 2. Graphical interpretation of hyperbox selection

where δ equals one-third of the side length of the cube ($\delta = \frac{1}{3}\varepsilon$) and e_i is the i^{th} Euclidean base vector. w_i is defined as the $min\{f(c-\delta e_i), f(c+\delta e_i)\}, 1 \leq i \leq N$, and the division is done in the order given by w_i, starting with the lowest w_i. Thus, the hyperbox is first partitioned along the direction of lowest w_i and then the remaining field is divided along the direction of the second lowest w_i and so on until the hyperbox is partitioned in all directions. From this point onwards, the algorithm starts identifying the potentially optimal hyper boxes, dividing these hyper boxes further and sampling at their centre points until the termination criteria is satisfied.

3.2 *Potentially optimal hyperboxes*

Suppose the unit hyper box is divided into m smaller hyper boxes. Let c_i denote the centre point of the i^{th} hyperbox and ε_i the distance from the centre point to the vertices. One box among these m hyper boxes must be selected for further sampling.

Definition *1.* (Potentially optimal hyper box). Let ξ be a positive constant and f_{min} be the current lowest function value. A hyper-rectangle/box j is said to be potentially optimal if there exists some constant rate of change $K > 0$ such that

$$f(c_j) - K\varepsilon_j \leq f(c_i) - K\varepsilon_i, \text{ for any i=1,..,m} \quad (4)$$

$$f(c_j) - K\varepsilon_j \leq f(min) - \xi|f_{min}| \quad (5)$$

Figure 2 illustrates the above definition further. The horizontal co-ordinate is the size of the hyperbox, which is the distance from the centre to the vertices of the box. This captures the goodness based on the amount of unexplored region in the search space. The vertical co-ordinate is the value of the cost function at the centre point of the particular hyperbox. This captures the goodness of the interval with respect to the local search, that is the goodness based on the known function values. Each point on the graph represents a hyperbox.

The first condition in the definition forces the hyper box to be on the lower right of the convex hull of the dots. Hyperboxes having low objective function values are inclined to fall on the convex hull of the set, as are (relatively) large hyperboxes. One of the largest hyperboxes is chosen for division. The second condition insists that the lower bound for the interval, based on the rate of change K, exceed the current best solution by a nontrivial amount. This condition prevents the algorithm from becoming too local in its orientation. In terms of Figure 2, it implies that some of the smaller intervals might not be selected. In this way, the groups of hyperboxes are larger, and consequently the iteration places a stronger emphasis on the value of the objective function at the centre point of the hyperbox, which biases the search locally. The parameter ξ was introduced to balance the local and global searches, (D.R.Jones *et al.*, 1993). In Figure 2, the point $(0, f_{min} - \xi|f_{min}|)$ changes the convex hull so that the hyperbox with smallest objective function value need not be potentially optimal. By this approach, more sampling is done in larger, unexplored hyperboxes. If $\xi = 0.01$, then the lower bound for the hyper box would have to exceed the current best solution by more than 1%. Previous studies indicate that a choice of value for ξ ranging from 10^{-3} to 10^{-7} generally provides the best results, and a value of ξ equal to 10^{-4} was used in this study. The hyperboxes represented by points on the lower right convex hull of this graph satisfy the above equations and are thus potentially optimal.

3.3 *Termination criterion*

As there is no a priori estimate of a precise or probable number of iterations or function evaluations required to compute the global optimum, an adaptive termination criterion has been used in the present study. The criterion used is dependent on improvement in the solution accuracy over a finite number of successive simulations. The algorithm terminates the search if there is no improvement on the best solution achieved above a defined accuracy level, here chosen as 10^{-6}, for a defined successive number of simulations, presently fixed at 300. If the accuracy level is reduced, the algorithm will terminate faster, but the quality of the solutions will deteriorate and the probability of getting trapped in a local solution will increase.

The DIRECT pseudo-code is given in Table 2. For more details about the algorithm and further performance comparisons with other algorithms on standard test problems, the reader is referred to (D.R.Jones *et al.*, 1993; Finkel and Kelley, 2004).

3.4 *Hybridisation of DIRECT*

The hybridisation attempts to overcome one of the main disadvantages of the DIRECT algorithm, namely its lack of fast convergence to solutions that are on the bounds of the uncertain parameter space due to the centre point sampling strategy.

For our analysis, the DIRECT algorithm is augmented with a local optimisation method using a simple hybridisation strategy. A constrained direct search

Table 2. DIRECT & H-DIRECT Algorithm

(1) Normalise the search space to a unit hyperbox.
(2) Sample the centre point c_1 of the hyperbox; Evaluate $f(c_1)$. Set $f_{min} = f(c_1)$, $m = 1$, $t = 0$ (iteration counter), and $TC = 0$ (Termination Counter)
(3) **While** Termination criterion not satisfied ($TC \leq 300$) **do**
    (a) Identify the set S of *'potentially optimal'* hyperboxes
    (b) Select any rectangle/box $j \in S$
    (c) Divide the box j as follows:
        (i) Identify the set I of dimensions with the maximum side length ε. Let δ equal one-third of this maximum side length ($\delta = \frac{1}{3}\varepsilon$).
        (ii) Sample the function at the points $c \pm \delta e_i$, $\forall i \in I$, where c is the centre of the box and e_i is the i^{th} unit vector.
        (iii) Divide the box j containing c into thirds along the dimension I, starting with the dimension with the lowest value of $w_i = min\{f(c \pm \delta e_i)\}$, and continuing to the dimension with the highest w_i. Update f_{min}, x_{min} and m.
    (d) Set $S = S - j$. If $S \neq \oslash$ **GO TO** STEP (b)
    (e) Set $t = t + 1$. Calculate improvement in f_{min} obtained from previous iteration. $TC = TC + 1$ if improvement $\leq 1e^{-6}$ in subsequent iterations, if not set $TC = 0$.
(4) **END of While** **End of DIRECT Algorithm.**

BEGIN Hybridisation
(5) Choose the solution x_{min} from STEP4, set $x_{initial} = x_{min}$ and execute '*patternsearch*' algorithm to refine the global solution. (This is particularly effective when some, or all x_{min} are on their bounds.) **End of H-DIRECT Algorithm.**

method based on pattern search is incorporated into the DIRECT algorithm using the MATLAB function *"patternsearch"*(*Genetic Algorithm and Direct search Toolbox User's Guide, Version 2*, 2005). The general pattern search proceeds by performing several iterations of coordinate searches. From the starting solution, identified as a candidate point, the points that lie one step size away along the coordinate vectors are tested for improvement. When a coordinate search around a candidate point fails to find improvement, the step size is halved, and another coordinate search is performed. Otherwise, if improvement is found, a new temporary candidate point is constructed and used as the basis of a coordinate search. If no improvement is found around the temporary candidate point, the search backtracks to the last candidate basis point. Otherwise, if improvement is found around the temporary candidate point, the actual candidate point is updated, and a new coordinate search is started. The pattern search terminates when the step size drops below some threshold fixed at $1e^{-3}$ presently. For more details of the patternsearch algorithm, the reader is referred to (*Genetic Algorithm and Direct search Toolbox User's Guide, Version 2, 2005)*.

The solution obtained from the DIRECT algorithm is considered as the initial candidate solution for the pattern-search method. This hybridized DIRECT / patternsearch algorithm is referred to as H-DIRECT, see also Table 2 for its pseudo code.

4. CLEARANCE RESULTS

The optimisation-based worst-case analysis procedure is implemented in the Matlab 2006A and Simulink 6.1 environments. The various uncertain parameters listed in Table 1 are used as the optimisation parameters. To normalise the optimisation variables, the parameter values supplied to the optimisation algorithm are multiplied by the proper scaling factor. Before starting a simulation, the respective entries of the uncertain variables in the XML database are accessed and modified with the new values provided by the optimisation algorithm. Once the simulation is finished, the cost function as given in Eqn.(1) is evaluated. The optimisation algorithm iterates identifying potential solutions and eventually converging to the global solution. However, to control the computational complexity we defined an adaptive termination criterion for the worst case analysis problem. Once the DIRECT optimisation provides a solution, the patternsearch is applied to refine the solution at each iteration.

The H-DIRECT algorithm took a total number of 1269 simulations, 949 for DIRECT alone and the remainder for the pattern-search algorithm. The normalised worst-case obtained is $[\Delta_{mass}, \Delta_{Ixx}, \Delta_{Iyy}, \Delta_{Izz}, \Delta_{Ixz}, \Delta_{x_{cog}}, \Delta_{y_{cog}}, \Delta_{z_{cog}}]$=$[-1, 1, 1, 1, -1, -1, 1, 1]$.

Figure 3 shows the reference, worst-case and nominal angle-of-attack responses for the RLV model. Note that the maximum error is around 4 degrees at the beginning of the trajectory and decreases as the simulation is run.

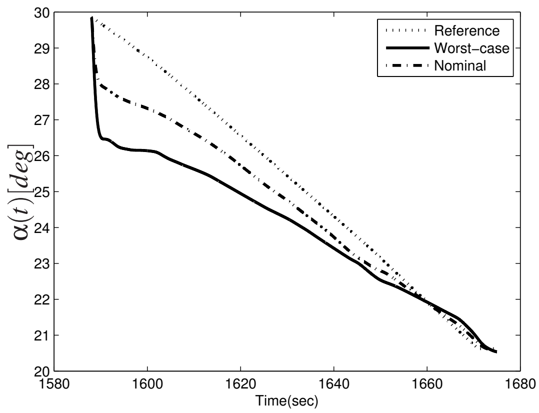

Fig. 3. Re-entry Angle of Attack trajectory

Figure 4 shows the corresponding nominal and worst-case deviations from the desired zero value of $\beta(t)$. Interestingly, although the present cost function depends only on the value of $\alpha(t)$, the significant amount of coupling between longitudinal and lateral dynamics at high AoA results in the worst-case β trajectory also being significantly different from the nominal response. To explore this issue further, the final version of the paper will include multi objective clearance criteria, to identify the set of worst-case uncertain parameters for all of the controlled variables that define

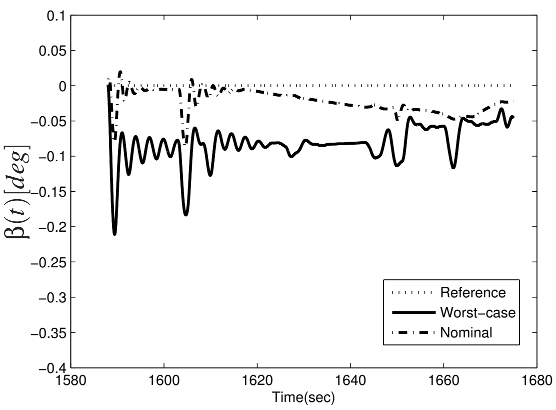

Fig. 4. Re-entry Angle of Sideslip trajectory

the reference trajectory. The fact that the worst-case value of the uncertainties describing mass, centre-of-gravity and inertia variations are all on their maximum or minimum bounds is not surprising, and agrees well both with flight mechanics intuition and with the results of previous studies. The situation becomes much more complex when stability derivatives, sensor errors, etc are included, however, the corresponding worst-cases do not necessarily lie on the uncertain parameter bounds.

In terms of computational complexity, we note that the total number of simulations required by the H-DIRECT algorithm was 1269. This compares badly with the $2^8 = 256$ simulations required to check all possible extreme points for the 8 uncertain parameters currently considered. However, when the number of uncertain parameters is increased (even to only 11 or 12), the total number of simulations required to check all extreme points rises (to 2048 and 4096 respectively) while the H-DIRECT number of simulation is not affected. Since we eventually expect to include up to 20 uncertain parameters for analysis, it is clear that the H-DIRECT algorithm has the potential to offer significant computational savings. To quantify further the computational benefits of the proposed approach, detailed comparisons will also be made with standard Monte-Carlo methods for worst-case analysis, as well as with hybridised global optimisation methods based on evolutionary computation.

5. CONCLUSIONS

In this paper, a novel hybridised version of the deterministic global optimisation algorithm, DIviding RECTangles (DIRECT), was applied to perform a worst-case analysis of a nonlinear-dynamic inversion (NDI) flight control law for a realistic simulation model of a re-entry vehicle over a particular phase of the trajectory for re-entry flight. A clearance criterion was defined based on the maximisation of the infinity norm of the error vector between the reference trajectory in Angle-of-Attack and the actual trajectory obtained by simulation of the model. Initial results of the study suggest that the proposed approach has the potential to improve significantly both the reliability and efficiency of the flight clearance process for future Reusable Launch Vehicles. The final version of the paper will include further analysis results using other clearance criteria, a more complete set of uncertain parameters, and further comparisons of the computational complexity of the proposed approach with respect to current industrial methods.

6. ACKNOWLEDGEMENTS

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