# Bridging the gap between linear and nonlinear worst-case analysis: an application case to the atmospheric phase of the VEGA launcher

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**Abstract:** This article presents the application of the structured singular value worst-case approach to the VEGA launcher during the atmospheric ascent phase. The analysis uses a linear fractional transformation model, formed from a subset of the uncertainty and dispersion parameters defined for the nonlinear VEGA system, to identify physically feasible worst-cases. This analysis is complementary to traditional Monte Carlo campaigns in that it provides an analytical guaranteed existence of worst-case conditions. To exemplify the difference, a dense (100,000 runs) Monte Carlo campaign in a high-fidelity VEGA nonlinear simulator yielded no performance-violating cases but, the results presented show that the application of mu-analysis is capable of identifying numerous parametric uncertainty combinations away from the extremes resulting in performance violations in the same VEGA nonlinear simulator at a fraction of the computational time. Thus, the present application shows that linear analytical approaches such as mu-analysis (when supported by proper development of a linear fractional transformation model) have capabilities that complement the traditional design verification and validation process for launchers.

*Keywords:* structured singular value, performance/robustness analysis, launcher systems

NOTATION & ACRONYMS

| | |
|---|---|
| ***Qα*** | Dynamic pressure times angle of attack criterion |
| ***LFT*** | Linear Fractional Transformation |
| **MC** | Monte Carlo |
| ***MCI*** | Mass-Center-Inertia properties |
| ***RACS*** | Roll and Attitude Control System |
| ***RP*** | Robust Performance |
| ***RS*** | Robust Stability |
| ***TVC*** | Thrust Vector Control |
| ***VNG*** | Non-gravitational velocity |
| ***V&V*** | Verification and Validation |

## 1. INTRODUCTION

The state-of-practice for launcher Verification & Validation (V&V) entails analysis of frequency domain requirements (i.e. stability margins) through a set of predefined vertex cases and, more critically, analysis of time domain requirements through nonlinear simulations in (i) a Monte Carlo setting (i.e. probabilistic approach) and (ii) using a set of selected worst-cases (i.e. deterministic approach).

Traditional MC campaigns consist of randomly sampling the uncertain parameters according to statistical distributions and deducing the values of criteria involved in the requirements. The number of simulations depends on specified probability and confidence levels. Although the MC approach is very practical in showing the design sensitiveness to parametric variations, it has many disadvantages in that it relies on massive amounts of computation without guaranteed proofs on the full parameter space and is questionable for maximum/minimum values analysis. Similarly, vertex approaches, whereby all the corner cases (min, max) are checked, are unfeasible whenever the dimension of the problem is relatively high. This is compensated by introducing engineering knowledge on the system which helps limit the number of corner cases to be examined. Nonetheless, the analysis is very limited and obviates altogether parameter combinations away from the extremes.

To overcome these limitations many advances have been explored in the field of aeronautical GNC V&V (Fielding et al 2002, Belcastro and Belcastro 2003, Jacklin et al 2006). One of the venues explored, which has proved very successful, has been the use of advanced optimization-based worst-case search algorithms (Storm and Price 1997, Menon et al 2009, Marcos et al 2013). The main downside of these approaches is the lack of guarantees in finding the worst-case, but they are very quick in identifying performance violation cases and their usage require only minimal adaptation (to the typical simulation models used in industry, e.g. Simulink/Matlab, Fortran, C-code). The other venue that has been followed in order to provide analytical guarantees is the use of analytical approaches such as the structured singular value μ (Balas et al 1998, Bateman et al 2005). The downside of μ-analysis is its linear nature (exemplified by its reliance on linear fractional transformation (LFT) models typically arising from linear dynamic systems), and the lack of associated probabilities to the identified worst-cases, which is necessary to provide a quantitative risk value.

Both types of approaches were considered in a European Space Agency (ESA) study led by Deimos and entitled "Robust Flight Control System Design Verification and Validation Framework (RFCS)" (Marcos et al 2011). This study was established with the objective of developing, demonstrating and comparing with a traditional V&V framework a new enhanced design V&V framework through their application to the V&V of a complex launch vehicle, specifically to the VEGA launcher.

In this article, the approach and results obtained from applying analytical μ in a frozen-time parametric uncertainty setting to VEGA during the 1st atmospheric phase are presented. It will be shown that the approach, which depends on proper development of a linear fractional transformation (LFT) model (Doyle et al 1991), can successfully find numerous performance violating cases valid not only in the nonlinear simulator used for verification but also in the official VEGA simulator used for validation. Furthermore, the model and analytical approach can be readily automated (with a view to further development of Monte Carlo guiding approaches or on-board analysis).

The layout of the paper is as follows: Chapter 2 presents the system and problem description. Chapter 3 presents a traditional Monte Carlo campaign as well as the results from applying optimization-based worst-case search algorithms. Chapter 4 presents the used μ analysis process and Chapter 5 presents the results .Chapter 6 ends with the conclusions.

## 2. VEGA LAUNCHER & NONLINEAR SIMULATORS

VEGA is the new European Small Launch Vehicle developed under the responsibility of ESA. The prime contractor for the launch vehicle is ELV. The launcher successfully performed a maiden launch at the beginning of 2012 from the Centre Spatial Guyanais in Kourou and a 2nd flight in May 2013.

VEGA follows a four-stage approach formed by 3 solid propellant motors (P80, Zefiro 23 and Zefiro 9) providing thrust for the 1st, 2nd and 3rd stages; and, a bi-propellant liquid engine (LPS) on the 4th stage. All four stages are controlled via a thrust vectoring system (TVC). There is also a Roll and Attitude Control System (RACS) performing 3-axes control during the ballistic phase and only roll rate control during the propelled phases.

The official non-real time high-fidelity nonlinear simulator used in the VEGA program is called VEGAMATH. This simulator is characterized by:

- High-fidelity 6 Degrees-of-Freedom motion
- Tail-Wag-Dog effects
- Bending and sloshing modes
- Full external environment (rotation Earth, winds…)
- Disturbances (torques at separation, bias, offsets)
- Nonlinear aerodynamics (incl. aero-elastic effects)
- TVC system (incl. computing delays, backlash, bias)
- Full representative code implementing the actual Guidance, navigation and control (GNC) system
- Propulsion and mass-center-inertia (MCI) properties
- Detailed inertial navigation system (INS)
- Detailed RACS models (thermal & thrust dynamics, filters, quantization and noise…)

The nonlinear simulator used in the RFCS project is called VEGACONTROL and is based on VEGAMATH but tailored to simulate the 1st atmospheric phase (P80) and not including bending/sloshing modes or the RACS model. This simulator is used by the RFCS consortium to develop and verify their approaches, and the results from these verifications are validated by ELV on the VEGAMATH.

Both simulators allow modifying the scattering values (uncertainties and dispersions) of up to 95 different uncertain parameters: e.g. MCI, aerodynamics, wind profiles, INS mounting and thrust among others. Each scattering variable is represented by a "flag" parameter with values ranging in the interval [-1, 1] with the zero value indicating "nominal".

## 3. MONTE CARLO & OPTIMIZATION-BASED ANALYSIS

A traditional Monte Carlo (MC) campaign was performed on VEGACONTROL using one hundred thousand (100,000) random combinations of the global uncertain set (i.e. the full 95 uncertain parameters). For analysis several robust performance indicators were postulated using a normalized multi-objective cost (see Marcos et al 2013) but no performance-violating cases was found (i.e. no cost function higher than 1.0 was found). Figure 1 shows the 11 cases that achieved a normalized cost higher than 0.9 (but <1.0) for the *Qα-versus-Mach* criterion, i.e dynamic-pressure times angle of attack versus Mach (one of the main load assessment criterion for launchers).

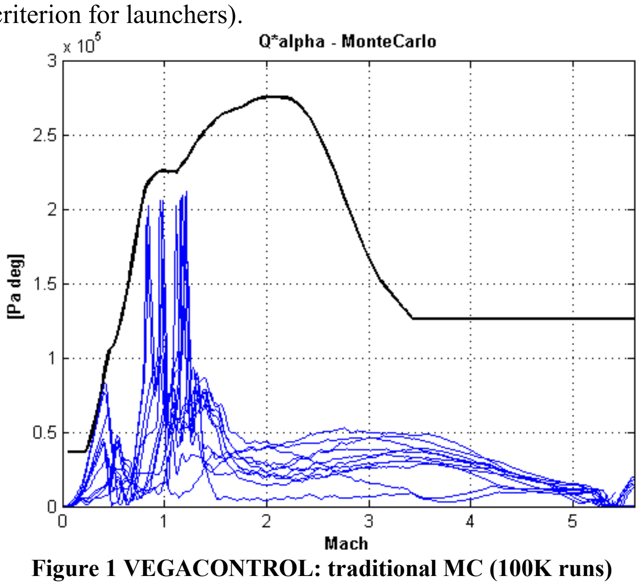


**Figure 1 VEGACONTROL: traditional MC (100K runs)**

In that same reference (Marcos et al 2013), a worst-case time-domain optimization toolbox WCAT (Menon et al 2009) was used to identify worst-cases on the VEGACONTROL. Using in an iterative manner a number of pre-defined parameters sets (e.g. 'aero-elastic', 'aerodynamics', 'MCI'…), 58 cases that violated one or several performance bounds were identified and verified in the VEGACONTROL (Figure 2). These cases were validated in VEGAMATH and showed a 'tight' connection between the results from both simulators.

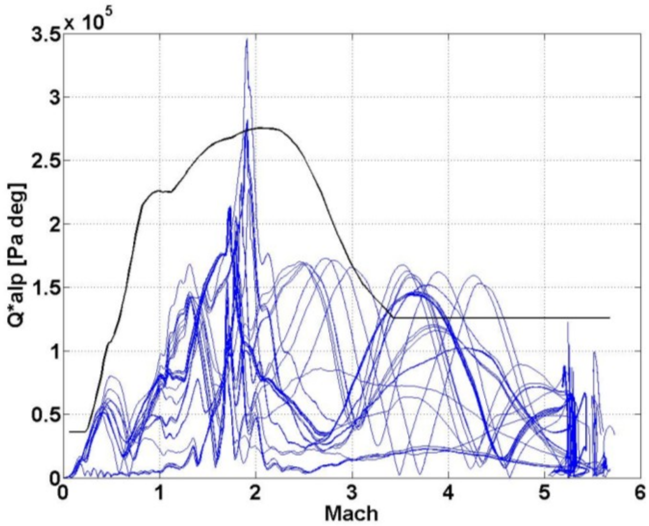


**Figure 2 VEGACONTROL: 58 WCAT worst-cases**

## 4. WORST-CASE MU ANALYSIS & LFT THEORY

A concept widely used in robust control is the structured singular value μ, which analytical evaluates the robustness of uncertain systems (Doyle et al 1991, Balas et al 1998). A key aspect on the application of μ is the development of a proper LFT model. By proper it is meant a model that captures the critical parametric behaviour of the nonlinear system under consideration. In this section, the theory behind μ analysis and LFT modeling is briefly reviewed.

### *4.1 Structured Singular Value Analysis*

The structured singular value $\mu_\Delta(M)$ of a matrix $M \in C^{n\times n}$ with respect to the uncertain matrix $\Delta$ is defined in Eq. (1), where $\mu_\Delta(M)=0$ if there is no $\Delta$ satisfying the determinant condition.

$$\mu_\Delta(M) = \frac{1}{\min_\Delta(\bar{\sigma}(\Delta) : \det(I - \Delta M) = 0)} \qquad (1)$$

Note that this definition is given in terms of an $\{M,\Delta\}$ model which is an LFT model where $\Delta$ is typically norm-bounded $\|\Delta\|_\infty \le 1$ (without loss of generality by scaling of $M$) for ease of calculation/interpretation. In this manner, if $\mu_\Delta(M) \le 1.0$ then the result guarantees that the analyzed system, represented by the LFT, is robust to the considered uncertainty level. The structured singular value is a robust stability (RS) analysis but can be used also for robust performance (RP) as this problem can be transformed very straightforward into a RS problem (Doyle et al 1991).

Since $\mu_\Delta(M)$ is difficult to calculate exactly, the algorithms implement upper and lower bound calculations (Balas et al 1998). The upper bound $\mu_{upper}$ provides the maximum size perturbation $|\Delta|_\infty = 1/\mu_{upper}$ for which RS/RP is guaranteed, while the lower bound $\mu_{lower}$ guarantees the minimum size perturbation $|\Delta|_\infty = 1/\mu_{lower}$ for which RS/RP is guaranteed to be violated. Thus, if the bounds are close in magnitude then the conservativeness in the calculation of $\mu$ is small, otherwise nothing can be said on the guaranteed robustness of the system for perturbations within $[1/\mu_{upper}, 1/\mu_{lower}]$

### *4.2 Linear Fractional Transformation Modeling*

An LFT is a representation of a system using two matrix operators, $M=[M11\ M12;\ M21\ M22]$ and $\Delta$, and a feedback interconnection. The matrix $M$ represents the nominal (known) part of the system while $\Delta$ contains the unknown, time-varying or uncertain parameters $\rho_i$. Depending on the feedback interconnection used, there are two possible types of LFTs: lower and upper (see Figure 3).

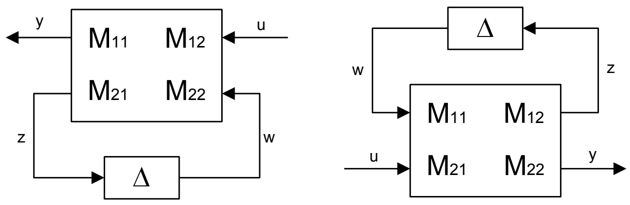


**Figure 3 Lower and upper LFTs**

The matrix $\Delta$ is unrestricted in form (structured or un-structured) or type (nonlinear, time-varying or constant). It is important to note that unstructured uncertainty at component level becomes structured at system level. The order of the LFT is the number of parameters, including repetitions, contained in $\Delta$ (e.g. $\Delta = diag\ (\rho_1 I_2, \rho_2)$ ➔ LFT of order 3 with 2 parameters $\rho_i$). Since the parameters are typically fitted in a polynomial fashion to the available data, many realistic robustness analysis problems easily result in very high order LFTs. Thus it is vital to have efficient (and automated) tools which can compute minimal, or at least close to minimal, representations of these systems. The selection of the variable set $\rho(t) \in \Delta$ that captures the behaviour of the nonlinear system is a task that is not always obvious a priori. Indeed, this step is key to obtain a reliable LFT that will yield relevant and meaningful results and despite its apparent simplicity is where most of the LFT modeling effort and ingenuity is focused.

There are several approaches that can be used to obtain a reliable LFT model (Doyle et al 1991, Lambrechts et al 1993, Balas et al 1998, Magni 2004, Marcos and Balas 2004, Marcos et al 2007 a/b). The specific approach used in the RFCS project is based on the developments from the later two references, which formalized a modeling methodology to transform a general linear parameter varying model into an LFT representation through the use of symbolic manipulations. An example of the use of this symbolic LFT approach to obtain a manageable-size LFT design model starting from an exact LFT representation is given in (Marcos et al 2006) for the modelling of an on-ground aircraft where the challenging effects of on-ground tyre forces and moments are considered.

## 5. MU ANALYSIS PROCESS & RESULTS

### *5.1 From equations of motion to LFT*

It was already highlighted that the first, and key, step to apply μ is to obtain a reliable LFT model. This is typically done using a simplified representation of the system and assessing the best selection of the parameters and their polynomial fitting to be included in the matrix $\Delta$.

For any launcher vehicle, its yaw or pitch rigid-body motion dynamics is completely described by its attitude (yaw $\psi$ or pitch $\theta$) and linear motion ($z$ or $y$) in a frame linked to the velocity of the reference trajectory. Under the assumption of a perfect axial-symmetric launch vehicle, the dynamic equations can be expressed in a $\{A, B, C, D\}$ state-space:

$$A=\begin{bmatrix}0 & 1 & 0 & 0\\ 0 & a_1 & a_3 & a_2\\ 0 & 0 & 0 & 1\\ 0 & a_4 & A_6 & a_5\end{bmatrix};\quad B=\begin{bmatrix}0\\ a_P\\ 0\\ K1\end{bmatrix};$$
$$C=\begin{bmatrix}0 & 0 & 1 & 0\\ 0 & 1 & 0 & -a_Z\\ 1 & 0 & -a_Z & 0\end{bmatrix};\quad D=\begin{bmatrix}0\\ 0\\ 0\end{bmatrix}; \qquad (2)$$

For the yaw rigid-motion, the model has 4 states ($z, \dot{z}, \psi, \dot{\psi}$), 3 outputs ($\Delta\psi, \dot{z}, z$) and 1 input (nozzle deflection angle), where the coefficients are given by:

$$L_a = S_{ref} q_{dyn} C_N \frac{1}{\alpha}; \qquad \mu_a = -L_a (x_{CP} - x_{CoG}) \frac{1}{J_{yy}};$$
$$a_1 = -L_a \frac{1}{m}\frac{1}{V_{rel}}; \qquad a_2 = -a_1 (x_{CP} - x_{CoG});$$
$$a_4 = -\mu_a \frac{1}{V_{rel}}; \qquad a_5 = -a_4 (x_{CP} - x_{CoG}); \qquad (3)$$
$$a_z = x_{INS} - x_{CoG}; \qquad a_P = -T_h \frac{1}{m};$$
$$a_3 = -acc + a_1 V_{rel}; \qquad A_6 = a_4 V_{rel};$$
$$K_1 = -T_h (x_{CoG} - x_{PVP_{ref}}) \frac{1}{J_{yy}};$$

The given variables are defined as: relative velocity *Vrel*, dynamic pressure $q_{dyn}$, thrust *Th*, total launch vehicle mass *m*, total y-axis moment of inertia *Jyy,* normal aerocoefficient $C_N$, longitudinal acceleration *acc*, angle of attack $\alpha$, x-coordinate center of pressure $x_{CP}$ and x-coordinate center of gravity $x_{CoG}$. All these variables can be represented as depending on time *t* or on non-gravitational speed *VNG*. The launcher dynamic model given by (Eq.2-Eq. 3) captures the main characteristics of the 1st atmospheric phase (P80) and is typically used to design the launchers' controllers for this phase.

After careful assessment of the variables above and the scatterings 'flags' in the VEGACONTROL simulator, several nonlinear simulations were performed to determine the 'best' (i.e. most representative of the effects) set of parameters for the LFT. Due to space limitations, and the analysis process orientation of this article, no detailed description of the LFT model or process is given. Nonetheless, in order to provide a basic understanding on the appropriateness of the obtained LFT, the selected parametric set is given by 6 scatterings 'flags' associated to 4 of the variables from Equation (3): combustion time (thru *Th*), air density (ρ thru $q_{dyn}$), normal aerodynamic coefficient ($C_N$) and x-coordinate center of pressure ($x_{CP}$). These 4 time-varying parameters are introduced in the LFT using linear and bilinear polynomial fits based on the 6 scatterings 'flags' (which will go to $\Delta$) plus symbolic nominal and deviation constants (assigned to *M*). The final order of the LFT is 31 and it is highlighted that it is a symbolic representation of the system in Eqs. (2)-(3).

### *5.2 Analysis Process*

The analysis process is as follows:

A. Starting from the symbolic LFT model, select the time along the P80 ascent trajectory at which the analysis is to be performed.

   This will result in a specific *VNG* at which the symbolic constants in the LFT matrix *M* are numerically evaluated. Subsequently, this numerical LFT can be model reduced to obtain an order of 22. Note that except for this latter numerical model reduction, the symbolic LFT is fixed and applicable to any launcher in atmospheric phase. Thus, it can be used in an automated or on-board fashion.

B. Close the above LFT system with the TVC controller + actuator + delays loop. The appropriate TVC controller is obtained based on data from ELV and depends on the specific *VNG* value.

C. Select the proper μ structure for the $\Delta$ matrix.

   The structure can be real, complex or a mix of both (Balas et al 1998). The advantage is that more efficient algorithms can be used for the last two types. Also, in this step it is where the RS or RP formulation is defined. The difference is that the performance input/output channels for the RP case are closed using a fictitious uncertainty structure of appropriate dimensions so that a RS problem is obtained.

D. Perform the μ analysis. This is straight forward based on the previous steps.

E. Examine and verify the results. This step serves to ensure that the upper/lower bounds are close but also to determine the validity of the perturbation (i.e. proper size, nature and one that results in actual violation in VEGACONTROL).

F. ELV to validate the results in VEGAMATH.

*5.3 Mu Analysis Set-Up & Results*

The following analyses and $\Delta$ structures are used in step C:

- RS & RP using a pure real $\Delta$ (with a full complex block for the fictitious uncertainty in the RS case).
- RS using a 1% complex $\Delta$ for each uncertainty parameter

Each of the three μ analyses above is performed across the P80 trajectory sweeping from 0 to 100 seconds every 5 seconds (i.e. a total of 20 points for each analysis). Further, the numerical values for the LFT *M* matrix are obtained not only based on the time instance but also setting the rest of the VEGACONTROL flags set to their minimal, nominal and maximum deviation. Thus, a total of 180 (=3x20x3) μ analyses are performed where each takes an average of approximately 7 seconds, yielding a total of 23 minutes to perform the whole set (and including visualization, verification and storing of results). Finally, note that there are 4 potential μ worst-case combinations ($\mu_{wc}$) that can be obtained from the 3 analyses. This is because for the 1% complex RS, two real worst-cases can be obtained due to the doubling of the $\Delta$ structure (i.e. real and complex parts).

Of the maximum of 240 $\mu_{wc}$ arising from the 180 μ analyses performed, there are a total of 99 with $|\Delta|<1.2$ (a slightly higher value to 1.0 is used to account for numerical issues). Of these 99 cases there are 24 resulting in a *Qα-versus-Mach* performance bound violation in VEGACONTROL.

As an example of the μ results, Figure 4 shows the $\mu_{upper}$ (red solid) and $\mu_{lower}$ (blue dashed) bounds for three time instances (from top to bottom at 5, 15, 50 seconds) and for the 3 μ analyses (from left to right 'RS-real', 'RS-complex' and 'RP-real'). An asterisk in the plots indicates which of these cases yielded a combination of uncertain parameters that violated the performance bounds when verified in VEGACONTROL.

Observe that it is always better to calculate RS $\mu_{lower}$ using complex uncertainty, i.e. notice the dashed lines in the middle column with respect to those in the left column. And in any case, for the time instances at 15 and 50 seconds, $\mu_{lower}>1.0$ indicating that the flag combination is going to be well within $|\Delta|<1.0$ (i.e. it is an interior point) and actually in these 3 cases they are performance violating cases.

Also, note that for the RP analyses (right-most column), since the problem is normalized and no attempt is made to shape the frequency of the error channels, μ is always around 1.0 and the analysis must focus on examining the 'closeness' between the bounds. Indeed, for the time instances at 5 and 15 seconds (with RP $\mu_{upper}$ of 1.0 and 1.25 respectively and $\mu_{lower}=1$) the resulting $\mu_{wc}$ violate the performance bound in the VEGACONTROL while for the case at 50secs (where the distance between bounds is very large) there is no violation.

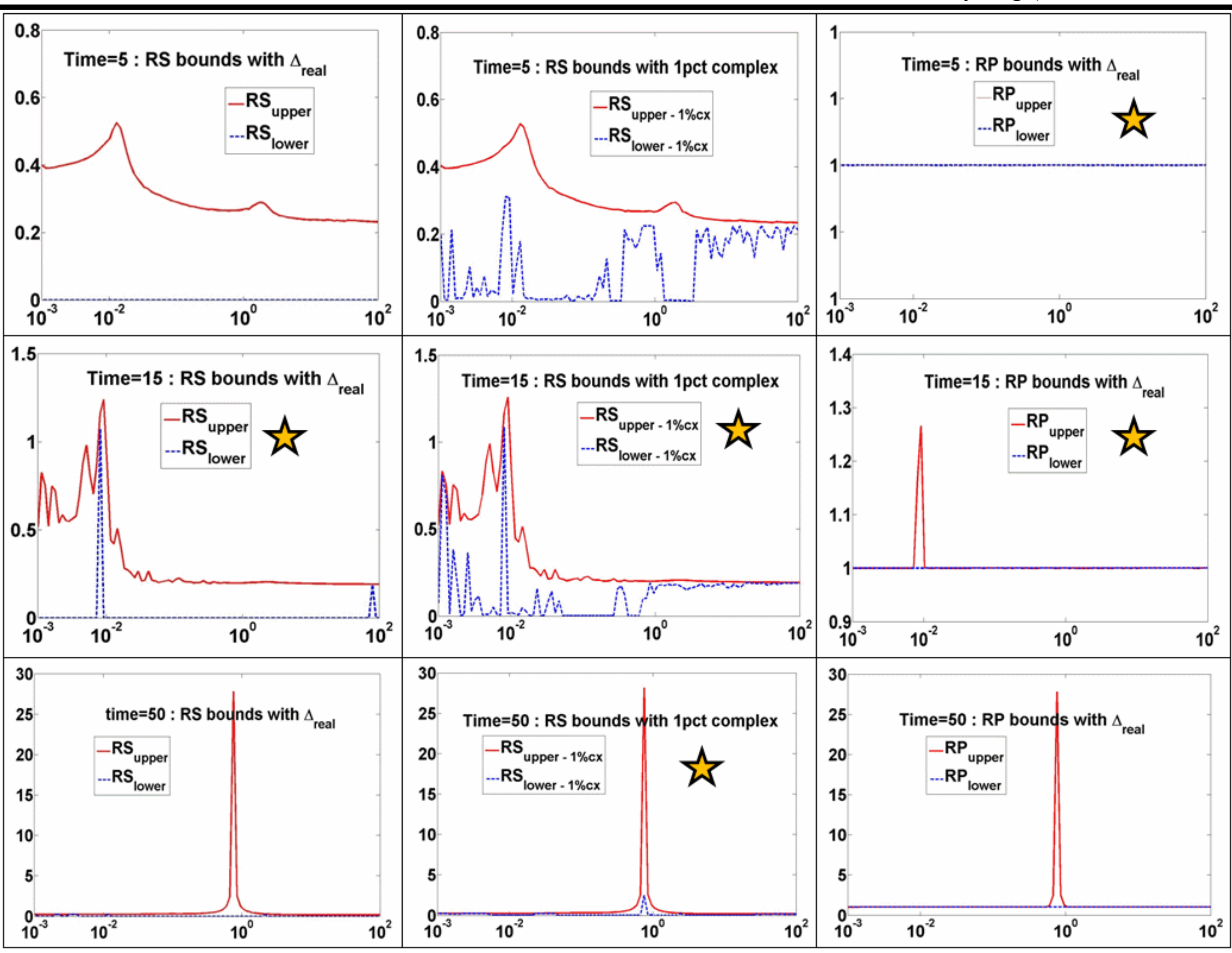


**Figure 4 μ worst-cases for (left-to-right) RS-real, RS-complex, RP-real at (top to bottom) time instance [5, 15, 50] seconds**

*5.4 Verification and Validation of Mu Results*

To conclude, Figure 5 shows the verification results obtained by applying the obtained 24 $\mu_{wc}$ to the VEGACONTROL simulator, while Figure 6 shows the validation of these cases on the VEGAMATH. Notice that all result in a violation of the bounds around similar Mach regions. Furthermore, the results are in agreement with those obtained by the application of WCAT (Figure 2).

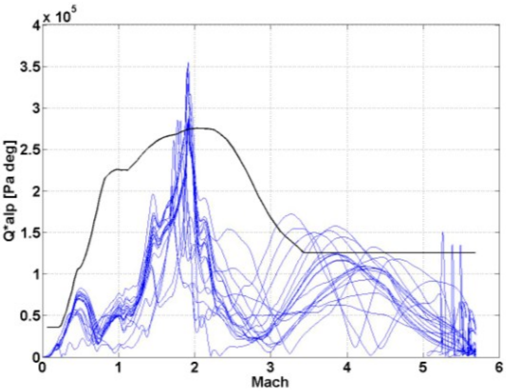


**Figure 5 VEGACONTROL: 24 μ worst-cases**

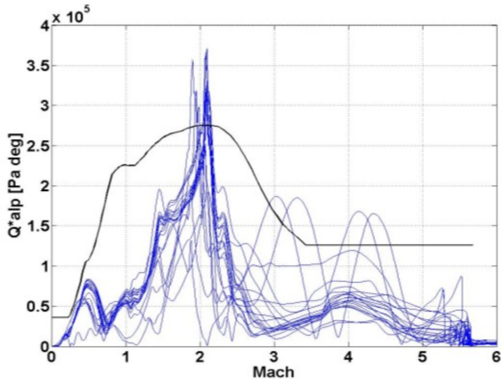


**Figure 6 VEGAMATH: 24 μ worst-cases**

## 6. CONCLUSIONS

It is well-known that traditional Monte Carlo campaigns, despite their use in industry and control certification, are not suitable for searching worst-case conditions or even to clear a parameter space. In the past 15 years, a shift to perform the clearance using optimization-based algorithms has occurred. These optimization-based approaches are highly efficient to identify de-stabilizing worst-cases even with as little as 1000 iterations and minor engineering knowledge (i.e. split of parameter space into subsets and setting other parameters to vertex values). Nevertheless, these approaches are not well connected to the design process nor are capable of guaranteeing the search for the full parameter space.

It is precisely due to the capability to address these last two concerns that the structured singular value μ has such a relevancy. There are always questions that the mismatch between the linear or pseudo-linear (LFT) models with the nonlinear system is too often too much, resulting in inconclusive analysis results. It has been shown in this article that it is possible for non-trivial cases to obtain a proper model that allows obtaining via μ analysis meaningful worst-case combinations guaranteed to violate the performance bounds on the high-fidelity simulators used for V&V activities, and at a fraction of the computational cost. Furthermore, since the obtained symbolic LFT is general (i.e. applicable to any launcher in atmospheric flight) the model and approach can be used for other launchers or in an automated fashion within a guided Monte Carlo algorithm.

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