Nested Integrated Control and Diagnostic Filter Design

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***Abstract*— In this paper an extension of the integrated approach to the concept of nested structures is given. This extension allows to iteratively optimize the closed loop for control and diagnosis performance. The advantage of the integrated approach is that the degradation of the diagnostic performance typically associated with changes in the controller is avoided. Furthermore, due to the iterative optimization possibility of the approach, the conflicting control and diagnostic objectives are optimally traded-off at each iteration.**

Typically, active fault tolerant systems imply the reconfiguration of the controller based on the information provided by a diagnostic filter. It is well-known that as the robustness and performance of the controller changes, the performance of the diagnostic filter might degrade to a point in which further fault information becomes unreliable (i.e. either it can not detect new faults or those detected might be false alarms). This can be avoided by reconfiguring the filter based on the reconfiguration of the controller or even on the information from the previous filter. The standard approach to address this problem is to design bank of diagnostic filters as well as of controllers, and choose the appropriate control and diagnostic filter depending on the detected abnormal situation [11], [7]. This implies extensive off-line design work as both the controllers and the filters are typically designed independently based on the assumed possible abnormal situations.

A similar approach could be developed, but avoiding the independent design of the controllers and diagnostic filters, by using the so-called integrated controller design approach [13], [6]. This would be advantageous as the integrated controller design approach is known to have several advantages with respect to the independent, non-integrated design of the control and diagnostic systems [9], [12]. Specifically: a) the conflicting objectives for the controller and the filter can be addressed directly in the integrated case, and b) the design coupling due to the system uncertainty can also be better traded-off. Nevertheless, this bank of integrated designs will still be dependent on the correct assumption on the plausible fault situations handled by the designs.

An additional advantage of the integrated framework is that it can be formulated within a linear fractional transformation (LFT) framework. This formulation allows to present the conflicting objectives in a clear fashion and also possibilitates the use of optimal techniques for the design of the $Q = [Q_c \;\; Q_f]$ free-parameter arising from the LFT representation [5]. More importantly, this LFT framework is also used in the nested structures proposed for the design of high-performance controllers in references [16], [15]. The key idea of nested structures is to characterize the plant and control systems as recursive LFTs which are based on successive designs of the Dual Youla S and Youla Q_c free-parameters, respectively. This succession of designs allow to improve the closed-loop properties by appropriate choice of the control Youla parameter Q_c based on the measured system uncertainty (incidentally given by the Dual Youla parameter S).

In this paper the idea of nested controllers is extended to the case of the integrated control approach. This extension facilitates the optimal joint design of the control and diagnostic systems based on the on-line information which arises from the measured uncertainty S or from the fault information provided by the filter free-parameter Q_f. An algorithm is proposed based on references [16], [15].

I. Integrated Controller

The integrated, or four-parameter, controller is a generalization of the Youla parameterization which extends the controller to four degrees of freedom (DoF) by considering reference signal tracking, closed-loop stabilization, residual generation, and disturbance rejection [8].

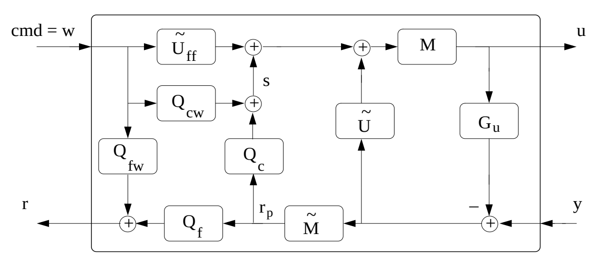

Fig. 1. Four-parameter controller parameterization structure.

Fig. 1 shows the structure for the parameterization of all four-parameter controllers given a system $G_u = N_u M^{-1} \in \mathcal{R}_P$ (the space of real-rational, and proper functions), and nominal feedback $K_o = \tilde{V}^{-1}\tilde{U}$ and feedforward $K_{ff} = \tilde{V}^{-1}\tilde{U}_{ff}$ controllers. Note, that the coprime factor $\tilde{V}^{-1}$ is hidden in Fig. 1 but can be "extracted" using the Bezout relation $\tilde{V}M - \tilde{U}N = I$ and the transfer function from w and y to u. Furthermore, the coprime factor $\tilde{V}$ is assumed common with the feedback controller K_o due to the well-known requirement that K_{ff} be stable or if unstable, implemented together with K_o. The general result for the four-parameter controller parameterization is formalized in the following theorem:

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Theorem I.1 (General Four-parameter Controller) *Consider a nominal plant $G_u \in \mathcal{R}_P$. Assume corresponding nominal stabilizing $K_o \in \mathcal{R}_P$ and feedforward $K_{ff} \in \mathcal{R}_P$ controllers are given. Let any right / left coprime factorization (r.c.f. / l.c.f.) for the nominal plant, $G_u = N_u M^{-1} = \tilde{M}^{-1}\tilde{N}_u$, and the controllers, $K_o = UV^{-1} = \tilde{V}^{-1}\tilde{U}$; $K_{ff} = U_{ff}V^{-1} = \tilde{V}^{-1}\tilde{U}_{ff}$, be known. The class of all proper integrated (stabilizing and residual generator) controllers $K_F(Q) \in \mathcal{R}_P$ is parameterized by:*

$$\begin{bmatrix} u \\ r \end{bmatrix} = \begin{bmatrix} (\tilde{V}+Q_c\tilde{N}_u)^{-1}(\tilde{U}+Q_c\tilde{M}) & (\tilde{V}+Q_c\tilde{N}_u)^{-1}(\tilde{U}_{ff}+Q_{cw}) \\ Q_f\left(\tilde{M}-\tilde{N}_u(\tilde{V}+Q_c\tilde{N}_u)^{-1}(\tilde{U}+Q_c\tilde{M})\right) & Q_{fw}-Q_f\tilde{N}_u(\tilde{V}+Q_c\tilde{N}_u)^{-1}(\tilde{U}_{ff}+Q_{cw}) \end{bmatrix} \begin{bmatrix} y \\ cmd \end{bmatrix} \tag{1}$$

Under the following conditions: $(\tilde{V}+Q_c\tilde{N}_u)(\infty)$ exist and

$$\begin{bmatrix} s \\ r \end{bmatrix} = Q \begin{bmatrix} r_p \\ cmd \end{bmatrix} = \begin{bmatrix} Q_c & Q_{cw} \\ Q_f & Q_{fw} \end{bmatrix} \begin{bmatrix} r_p \\ cmd \end{bmatrix} \in \mathcal{RH}_\infty, \tag{2}$$

where u is the feedback control input, r the residual vector, y the plant measurements and $cmd = w$ the exogenous input. The internal signals s and r_p are respectively the contribution of the Youla parameters to the feedback controller and the primary residual.

In [5] it is shown that the above general integrated controller parameterization can be formulated as a lower fractional transformation $K_F(Q) = F_l(\tilde{J}, Q)$, where the free matrix Q is given by (2) and the coefficient matrix $\tilde{J}$ by:

$$\begin{bmatrix} u \\ r \\ r_p \\ cmd \end{bmatrix} = \begin{bmatrix} \tilde{V}^{-1}\tilde{U} & \tilde{V}^{-1}\tilde{U}_{ff} & \tilde{V}^{-1} & 0 \\ 0 & 0 & 0 & I \\ V^{-1} & -\tilde{N}_u\tilde{V}^{-1}\tilde{U}_{ff} & -\tilde{N}_u\tilde{V}^{-1} & 0 \\ 0 & I & 0 & 0 \end{bmatrix} \begin{bmatrix} y \\ cmd \\ s \\ r \end{bmatrix} \tag{3}$$

The LFT formulation of the integrated controller allows for a more transparent look at the interactions between the different objectives: control and diagnostic. It was also shown that this LFT of the integrated controller is a general representation of specialized controller architectures widely used in the fault tolerant & diagnostic community.

II. Nested Structures

The idea of nested structures arises from considering LFT representations for the controller $K(Q_c)$ and the plant $G(S)$ which are assumed to be composed of two parts: a nominal part, K and G, and a robustifying Q_c or uncertain S part (depending whether the controller or the plant is under consideration). The last terms represent the Δ matrix from the standard $M-\Delta$ representation of an LFT [10], see Fig. 2 for the graphical form of an upper LFT $y = F_u(M,\Delta)u = [M_{22} + M_{21}(I-\Delta M_{11})^{-1}\Delta M_{12}]u$.

The development of the nested structures is based on the well-known Youla parameterization for the controller, $K(Q_c) = F_l(J_K, Q_c) = (\tilde{V}+Q_c\tilde{N}_u)^{-1}(\tilde{U}+Q_c\tilde{M}) = U_Q V_Q^{-1}$, and the so-called Dual Youla parameterization for the plant $G(S)$. The latter provides the parameterization for the family of plants stabilizable by a given controller. Its representation is similar to the Youla parameterization and can be given

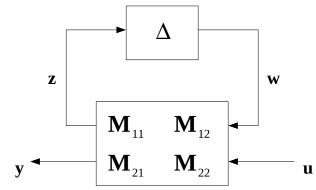

Fig. 2. Upper linear fractional transformation.

by an upper LFT $G(S) = F_u(J_G, S)$ between the factorization factors of the nominal plant G_u and a free parameter S which accounts for the plant uncertainty [15]:

$$\begin{aligned} G(S) &= (N_u + V_Q S)(M + U_Q S)^{-1} \\ &= N_u M^{-1} + \tilde{M}^{-1} S (I + M^{-1} U_Q S)^{-1} M^{-1} \end{aligned} \tag{4}$$

It is revealing to represent S in terms of the nominal plant G_u and the family of plants $G(S) = N(S)M(S)^{-1}$:

$$S = \tilde{M}(G(S)-G_u)M(S) = \tilde{M}(S)(G(S)-G_u)M \tag{5}$$

An interpretation of S can then be given as a frequency weighted measure of the difference between the nominal plant and the actual plant. Note that the uncertain term S can be similarly parameterized based on successive approximations of the uncertainty S_i with $i = 1, 2, \ldots, n$.

These LFT representations for the Dual Youla and the uncertain parameters S_i allow to represent any plant in an n-recursive LFT fashion whereby each successive plant $Fu(J_{Gi}, S_i)$ is a closer approximation to the real model, see Fig. 3(a).

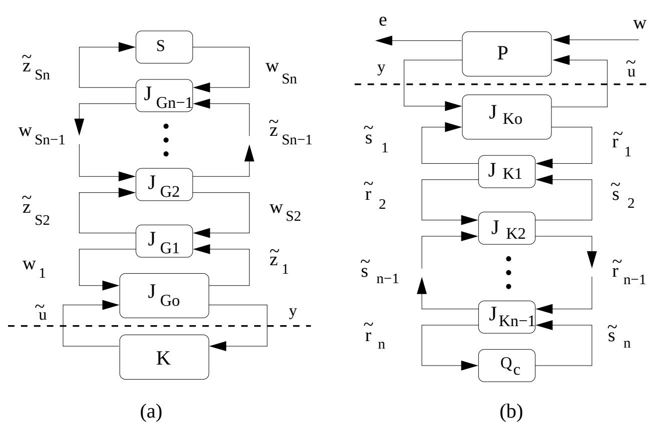

Fig. 3. Nested plants and controllers representations.

A similar result is valid for any controller where each successive design (i.e. J_{Ki} based on Q_{c_i}) improves the performance of the closed loop based on the actual measured uncertainty S_i, see Fig. 3(b). The system $P = \left[\begin{smallmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{smallmatrix}\right]$ represents a generalized plant whose nominal component is $P_{22} = G_u$.

These results form the basis of the multi-controller structures known as nested controllers. They are formalized by the following two lemmas taken from references [16], [15]:

Lemma II.1
Given a sequence of plants $G_i = N_i M_i^{-1} \in \mathcal{R_P}$ and corresponding strictly proper stabilizing controllers $K_i = U_i V_i^{-1} \in \mathcal{R_P}$ with $i = 0,1,...,n-1$. Then any transfer function $\bar{G} \in \mathcal{R_P}$ can be expressed in the following recursive manner in terms of a unique $G_n = S \in \mathcal{R_P}$ and noting $\bar{G} = G_o(S)$:

$$G_i(S) = \left(N_i + V_i G_{i+1}(S)\right)\left(M_i + U_i G_{i+1}(S)\right)^{-1} \tag{6}$$

for $i = 0,1,...,n-1$. Conversely, any given $G_n = S \in \mathcal{R_P}$ yields recursively $G_{n-1}(S)$,$G_{n-2}(S)$ $\ldots$ $G_o = G(S) \in \mathcal{R_P}$.

Lemma II.2
Given a transfer function $\bar{G} = G(S) \in \mathcal{R_P}$ with the recursive representation of (6). And given a series of double coprime factorization factors $N_i, M_i, U_i, V_i \in \mathcal{RH}_\infty$ such that N_i, U_i are strictly proper. Then any rational proper controller $\bar{K} \in \mathcal{R_P}$ for $\bar{G} = G(S)$ can be parameterized recursively in terms of a unique $K_n = Q_c \in \mathcal{R_P}$ (where $\bar{K} = K_o(Q_c)$):

$$K_i(Q_c) = \left(U_i + M_i K_{i+1}(Q_c)\right)\left(V_i + N_i K_{i+1}(Q_c)\right)^{-1} \tag{7}$$

for $i = 0,1,...,n-1$.

The above nesting of the controller and plant –based on their respective Youla and Dual Youla parameterizations– has the advantage of providing a reconfigurable controller strategy based on the design of $K_i(Q_c)$ in the face of an identified $G_i(S)$. This S-identification / Q_c-design can be carried out iteratively, or from a bank of Q_c controllers, where each Q_c improves the closed-loop performance/robustness characteristics.

In order to practically apply these concepts it is necessary to ascertain that the arbitrariness on the choice of plant and controller does not introduce instabilities in the nested structure. This was proven in references [16], [15] as well:

Theorem II.1
Assume P is stabilizable with respect to the general feedback interconnection in Fig. 4 where $K = K_o = UV^{-1}$ represents the nominal controller in $K(Q_c)$. Then the multiple control system in Fig. 3(b) with P replaced by $P(S)$ is internally stable if and only if $Q_c \in \mathcal{R_P}$ stabilizes $S \in \mathcal{R_P}$.

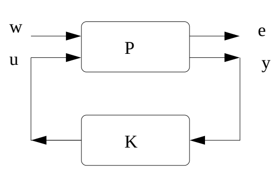

Fig. 4. General Feedback Interconnection.

Note that the free-parameters Q_c and S are not restricted to the stable Hilbert space $\mathcal{RH}_\infty$ in the above theorem.

III. Nested Extension for the Integrated Control

In this section, the extension of the integrated controller to nested structures is provided. As mentioned before, there are several integrated controller architectures available for implementation [14], [13], [17]. We will use the integrated controller architecture known as the Residual Generation - Generalized Internal Model Control (RG-GIMC) [5], see Fig. 5. This architecture is based on a variant of the Youla parameterization which allows for a separation principle on the controller and employs a residual generation free-parameter Q_f which is connected to the frequency domain residual generator parameterization from [2].

The extension of the integrated controller to nested structures results in two main benefits: first, due to the integrated design, the approach results in an improved capability to address the trade-off between the control objectives (by Q_c) and the diagnostic performance (by Q_f), i.e. through the direct design of $Q = [Q_c \;\; Q_f]$. Second, improvements on the closed-loop performance/robustness can be based on the successive S-identification / Q-design.

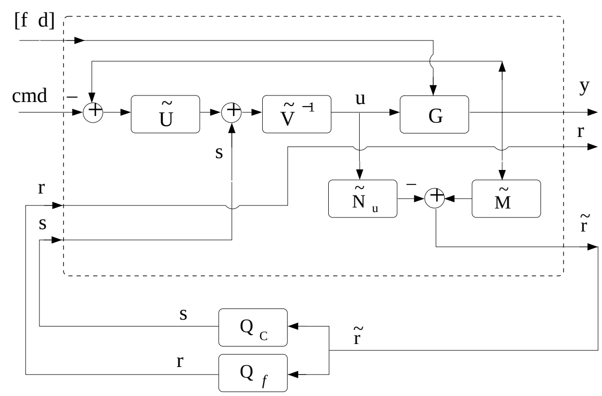

Fig. 5. Standard RG-GIMC paradigm.

The first step is to especialize the result from equation (1) to the class of RG-GIMC controllers $K_{RG}(Q)$ considered [5]:

$$\begin{bmatrix} u \\ r \end{bmatrix} = \begin{bmatrix} (\tilde{V}+Q_c\tilde{N}_u)^{-1}(\tilde{U}+Q_c\tilde{M}) & -(\tilde{V}+Q_c\tilde{N}_u)^{-1}\tilde{U} \\ Q_f\left(\tilde{M}-\tilde{N}_u(\tilde{V}+Q_c\tilde{N}_u)^{-1}(\tilde{U}+Q_c\tilde{M})\right) & Q_f\tilde{N}_u(\tilde{V}+Q_c\tilde{N}_u)^{-1}\tilde{U} \end{bmatrix} \begin{bmatrix} y \\ cmd \end{bmatrix} \tag{8}$$

The associated coefficient matrix for its lower LFT $K_{RG}(Q) = F_l(J_{RG-GIMC}, Q)$ is then given by $J_{RG-GIMC}$:

$$\begin{bmatrix} u \\ r \\ r_p \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} \tilde{V}^{-1}\tilde{U} & -\tilde{V}^{-1}\tilde{U} \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} \begin{bmatrix} \tilde{V}^{-1} & 0 \end{bmatrix} \\ \begin{bmatrix} 0 & I \end{bmatrix} \end{bmatrix} \\ \begin{matrix} V^{-1} & \tilde{N}_u\tilde{V}^{-1}\tilde{U} \end{matrix} & -\begin{bmatrix} V^{-1}N_u & 0 \end{bmatrix} \end{bmatrix} \begin{bmatrix} y \\ cmd \\ s \\ r \end{bmatrix} \tag{9}$$

where $Q = [Q_c \;\; Q_f]^\top \in \mathcal{RH}_\infty$ and $(\tilde{V}+Q_c\tilde{N}_u)(\infty)$ is invertible.

It is straight-forward (although a bit tedious) to obtain the nominal RG-GIMC closed-loop transfer function $TF_{\tilde{w}\to\tilde{e}}$ for the generalized plant P:

$$\begin{bmatrix} e \\ r \end{bmatrix} = \begin{bmatrix} P_{11}+P_{12}M(\tilde{U}+Q_c\tilde{M})P_{21} & -P_{12}M\tilde{U} \\ Q_f\tilde{M}P_{21} & 0 \end{bmatrix} \begin{bmatrix} w \\ cmd \end{bmatrix} \tag{10}$$

The next step pertains the stability equivalence given in Theorem II.1 especialized to the RG-GIMC controller

$K_{RG}(Q)$, an uncertain plant $G(S)$ (instead of P), and their LFT formulations:

Theorem III.1
Let $(G_u, K_{RG}(Q))$ be a stabilizing pair with coprime factorizations given by $G_u = N_u M^{-1} = \tilde{M}^{-1}\tilde{N}_u$ and (8) respectively. Let $G(S)$ have the factorization given in (4). Then, the pair $(G(S), K_{RG}(Q))$ is stabilizing if and only if the pair (S, Q) is stabilizing, see Fig. 6.

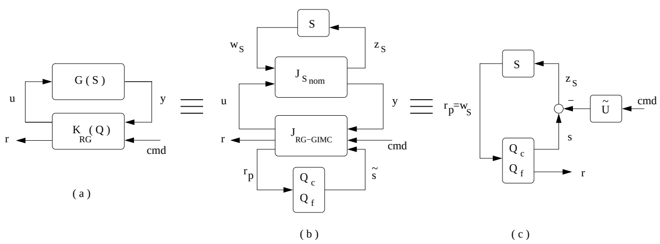

Fig. 6. Closed-loop $(G(S), K_{RG}(Q)) \equiv (S, Q)$.

Proof: First we need to prove that the pair $(G(S), K_{RG}(Q))$ is stabilizing. This follows the proof of Theorem 3.1 in [5] noting that the controller and the plant are given now by:

$$\begin{bmatrix} e \\ y \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & G(S) \end{bmatrix} \begin{bmatrix} w \\ u \end{bmatrix}; \quad \begin{bmatrix} u \\ r \end{bmatrix} = \begin{bmatrix} K_{11}(Q_C) & 0 \\ K_{21}(Q_C, Q_f) & 0 \end{bmatrix} \begin{bmatrix} y \\ cmd \end{bmatrix} \tag{11}$$

Then, the only stability conditions that remain are:

$$\begin{bmatrix} I & -K_{11}(Q_c) \\ -G(S) & I \end{bmatrix}^{-1} \in \mathcal{RH}_\infty; \tag{12}$$

$$\begin{bmatrix} I & 0 \\ 0 & G(S) \end{bmatrix} (I - K_{11}(Q_c)G(S))^{-1} K_{12}(Q_c) \in \mathcal{RH}_\infty \tag{13}$$

Recall that the second condition indicates the requirement that any unstable pole of K_{12} be implemented together with K_{11}. The first condition has been proven in Theorem 4-4.2, page 76 of reference [15] to be stabilizabled iff (S, Q_c) is a stable pair. The proof is easily obtained by proving the equivalence of the closed-loops in Figure 6. This equivalence follows from the combination of the LFT coefficient matrix $J_{RG-GIMC}$ from equation (9) with the LFT coefficient from the Dual-Youla parameterization of $G(S)$ from (4), $J_{S_{nom}}$:

$$\begin{bmatrix} z_S \\ y \end{bmatrix} = J_{S_{nom}} \begin{bmatrix} w_S \\ u \end{bmatrix} = \begin{bmatrix} -\tilde{U}\tilde{M}^{-1} & M^{-1} \\ \tilde{M}^{-1} & \tilde{M}^{-1}\tilde{N}_u \end{bmatrix} \begin{bmatrix} w_S \\ u \end{bmatrix} \tag{14}$$

which yields the combined coefficient matrix J_T:

$$\begin{bmatrix} z_S \\ r \\ r_p \end{bmatrix} = \begin{bmatrix} J_{T_{11}} & J_{T_{12}} & J_{T_{13}} & 0 \\ 0 & 0 & 0 & I \\ J_{T_{31}} & J_{T_{32}} & J_{T_{33}} & 0 \end{bmatrix} \begin{bmatrix} w_S \\ cmd \\ s \\ r \end{bmatrix} \tag{15}$$

where the coefficients are obtained after manipulating the signals from (9) and (14):

$$J_{T_{11}} = -\tilde{U}\tilde{M}^{-1} + M^{-1}(I - K_o G)^{-1} K_o \tilde{M}^{-1} = 0 \tag{16}$$
$$J_{T_{12}} = -M^{-1}(I - K_o G)^{-1} K_o = -\tilde{U} \tag{17}$$
$$J_{T_{13}} = M^{-1}(I - K_o G)^{-1} \tilde{V}^{-1} = I \tag{18}$$
$$J_{T_{31}} = V^{-1}(I - GK_o)^{-1} \tilde{M}^{-1} = I \tag{19}$$
$$J_{T_{32}} = \tilde{N}_u K_o - V^{-1}(I - GK_o)^{-1} G K_o = 0 \tag{20}$$
$$J_{T_{33}} = V^{-1}(I - GK_o)^{-1} G\tilde{V}^{-1} - V^{-1} N_u = 0 \tag{21}$$

It is now straight-forward to obtain the diagram from Figure 6(c) and establish the stability result of (S, Q_c) for the case with no commands (direct also for $cmd \neq 0$). ■

In order to extend the above result to the more general case $P(S)$ from (22), as required in Theorem II.1, and to provide more insight into the effect the uncertainty has on the design of the integrated controller, the uncertain closed-loop transfer function is obtained next:

$$\begin{bmatrix} e \\ y \end{bmatrix} = P(S) \begin{bmatrix} w \\ u \end{bmatrix} = \begin{bmatrix} P_{11}(S) & P_{12}(S) \\ P_{21}(S) & P_{22}(S) \end{bmatrix} \begin{bmatrix} w \\ u \end{bmatrix} \tag{22}$$

Lemma III.1
The uncertain closed-loop for the RG-GIMC architecture, assuming $P(S)$ is admissible and $P_{22}(S) = G(S)$ is given by (4), can be represented by a lower linear fractional transformation formed by the Youla matrix $Q = [Q_c \ \ Q_f]^\top$ and the coefficient matrix $T_{\Delta_{RG-GIMC}}$, see Fig. 7:

$$\begin{bmatrix} \begin{bmatrix} e \\ r \end{bmatrix} \\ r_p \end{bmatrix} = \begin{bmatrix} T_{\Delta_{11}} & T_{\Delta_{12}} \\ T_{\Delta_{21}} & T_{\Delta_{22}} \end{bmatrix} \begin{bmatrix} \begin{bmatrix} w \\ cmd \end{bmatrix} \\ \begin{bmatrix} s \\ r \end{bmatrix} \end{bmatrix} \tag{23}$$

where

$$T_{\Delta_{11}} = \begin{bmatrix} P_{11}(S) + P_{12}(S)M(S)\tilde{U}P_{21}(S) & -P_{12}(S)M(S)\tilde{U} \\ 0 & 0 \end{bmatrix} \tag{24}$$

$$T_{\Delta_{12}} = \begin{bmatrix} P_{12}(S)M(S) & 0 \\ 0 & I \end{bmatrix} \tag{25}$$
$$T_{\Delta_{21}} = \begin{bmatrix} \tilde{M}(S)P_{21}(S) & 0 \end{bmatrix} \tag{26}$$
$$T_{\Delta_{22}} = \begin{bmatrix} S & 0 \end{bmatrix} \tag{27}$$

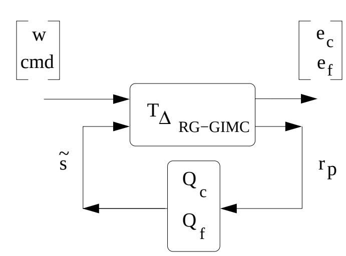

Fig. 7. Closed-loop $(T_{\Delta_{RG-GIMC}}, Q)$.

Furthermore, the uncertain closed loop for the RG-GIMC architecture $TF_{\tilde{w}\to\tilde{e}}$, given by (28), is stable if and only if

(G_u, K_o) *and* (S, Q) *are stabilizing pairs.*

$$\begin{bmatrix} e \\ r \end{bmatrix} = \begin{bmatrix} P_{11}(S) + P_{12}(S)M(S)\bigtriangledown_{Q_c} P_{21}(S) \\ Q_f\left(I + S(I - Q_c S)^{-1}Q_c\right)\tilde{M}(S)P_{21}(S) \end{bmatrix.$$
$$\begin{matrix} -P_{12}(S)M(S)\tilde{U} \\ 0 \end{matrix}\Bigg] \begin{bmatrix} w \\ cmd \end{bmatrix} \tag{28}$$

where $\bigtriangledown_{Q_c} = \left(\tilde{U} + (I - Q_c S)^{-1} Q_c \tilde{M}(S)\right)$.

Proof: See proof of Lemma 6.4.3 in reference [4]. ■

Remark 1. Note that the (2,2)-term of the uncertain coefficient matrix, equation (27), contains now the uncertainty description S as opposed to the corresponding term from the nominal case [5]. This is expected and is independent of the controller or plant used [15].

Remark 2. A comparison of the closed-loop transfer functions for the uncertain case, equation (28), and the nominal case, equation (10), yields that the residual generation channel is coupled to both Q-parameters for the former case (as opposed to the nominal case where both channels are only dependent on one of the Q parameters). This is in agreement with the results from reference [12].

The same lemmas as in the one-degree-of-freedom controller case, Lemmas II.1 and II.2, can be used to provide the nested recursion of the RG-GIMC coefficient matrices of the plant (9) and the controller (14). This is straightforward to prove by induction.

IV. Nested Integrated Control/Filter Algorithm

In this section an algorithm is proposed which uses the nested integrated approach to improve the closed-loop robustness under an abnormal situation (either fault or large uncertainty). It is based on the pseudo-code given in Chapter 5.3 of reference [15] and was outlined in [5]. An important assumption for the applicability of the algorithm is the availability of an identification scheme that estimates the plant uncertainty S, for example that from Hansen [3].

Two types of faults are typically considered in fault diagnosis: parametric and additive faults. The identification of the Dual Youla parameter S can be used to diagnose the parametric faults. An example of an FDI filter that estimates parametric faults is given in [1]. In certain cases, the integrated residual generator Q_f can also be used for this task. For simplicity, we will assume that the residual parameter Q_f in the integrated controller is to be designed to optimize the detection and isolation of additive faults and severe disturbances only.

The idea of the algorithm is to re-design (or select from a bank of designs) the free-matrix $Q = [Q_c \;\; Q_f]^\top$ to improve the robustness of the closed-loop in the face of faults (additive or parametric), severe disturbances, and/or uncertainty. At each re-design step, the performance of the control and diagnosis objectives might be reduced but the free parameters are chosen to maximize them for the new closed loop.

If parametric faults or large uncertainties are detected:

1.a. Given the nominal controller $K_o = U_o V_o^{-1}$ and plant $G_o = N_o M_o^{-1}$. Assume the true plant $\bar{G}$ is given, in terms of the unknown uncertainty S_o and the nominal plant G_o, using a Dual Youla parameterization:

$$\bar{G} = G_o(S_o) = \left(N_o + V_o S_o\right)\left(M_o + U_o S_o\right)^{-1} \tag{29}$$

1.b. Find an estimate of S_o and factorize it, i.e. find $\hat{S}_1 = N_{S_1} M_{S_1}^{-1}$. Use this estimate to define a new, more exact, and known 'nominal' plant G_1:

$$\begin{aligned} G_1 =& G_o(\hat{S}_1) = \left(N_o + V_o\hat{S}_1\right)\left(M_o + U_o\hat{S}_1\right)^{-1} \\ =& \left(N_o M_{S_1} + V_o N_{S_1}\right)\left(M_o M_{S_1} + U_o N_{S_1}\right)^{-1} \\ =& N_1 M_1^{-1} \end{aligned} \tag{30}$$

1.c. Design (or select) a new Q-controller by applying the integrated Youla/Dual-Youla ideas. In exactitude, synthesize (or select) Q_1 based on $\hat{S}_1$ such that: *i*) $(Q_{c_1}, \hat{S}_1)$ is a stabilizing and performance-optimal pair, *ii*) (Q_{c_1}, S_o) is a stabilizing pair, and *iii*) the trade-off for (Q_{c_1}, Q_{f_1}) is optimal.
Hence, the new 'nominal' controller $K_o(Q_{c_1})$ still maintains stability for the true plant but results in improved performance for G_1.

1.d. For the new controller free-parameter $Q_1 = U_{Q_1} V_{Q_1}^{-1}$, find an updated representation of the true uncertainty S_o based on the estimated $\hat{S}_1$ and a new unknown parameter S_1:

$$S_o = \hat{S}_1(S_1) = \left(N_{S_1} + V_{Q_1} S_1\right)\left(M_{S_1} + U_{Q_1} S_1\right)^{-1} \tag{31}$$

1.e. Find the new integrated controller $K_1 = [K_{c_1} \; K_{f_1}]^\top = U_1 V_1^{-1}$ using the parameterizations given in (8):

$$\begin{aligned} K_{c_1} =& K_o(Q_{c_1}) = \left(U_o + M_o Q_{c_1}\right)\left(V_o + N_o Q_{c_1}\right)^{-1} \\ =& \left(U_o V_{Q_{c1}} + M_o U_{Q_{c1}}\right)\left(V_o V_{Q_{c1}} + N_o U_{Q_{c1}}\right)^{-1} \end{aligned} \tag{32}$$

$$K_{f_1} = K_o(Q_{f_1}, Q_{c_1}) = Q_{f_1}\left(I + V_o^{-1} N_o Q_{c_1}\right)^{-1} V_o^{-1} \tag{33}$$

1.f. Find the new representation of the true plant, using again the Dual Youla parameterization from (4):

$$\begin{aligned} \hat{G} =& G_o(S_o) = G_o(\hat{S}_1(S_1)) = G_1(S_1) \\ =& \left(N_1 + V_1 S_1\right)\left(M_1 + U_1 S_1\right)^{-1} \end{aligned} \tag{34}$$

which can be rewritten using (30) and (32) as follows:

$$\hat{G} = \left(\left(N_o M_{S_1} + V_o N_{S_1}\right) + \left(V_o V_{Q_1} + N_o U_{Q_1}\right) S_1\right) \left(\left(M_o M_{S_1} + U_o N_{S_1}\right) + \left(U_o V_{Q_1} + M_o U_{Q_1}\right) S_1\right)^{-1} \tag{35}$$

1.g. Given the new pair $G_1 = N_1 M_1^{-1}$ and $K_1 = U_1 V_1^{-1}$ repeat from step 2: find a new estimate $\hat{S}_2$ of the uncertainty S_1, design new Q_2 such that $(Q_2, \hat{S}_2)$ is stable and optimal and $(Q_2, \hat{S}_1)$ stabilizing, and subsequently find the new plant and controller. Terminate when the closed-loop properties are acceptable.

If <u>additive faults</u> are detected:

2.a. Find an integrated controller, $Q_1 = U_{Q_1} V_{Q_1}^{-1}$ that stabilizes the system and provides optimal closed-loop and diagnostic performance.

2.b. Find the new controller, $K_1 = K_o(Q_1)$, given by (32) and (33).

2.c. Repeat if new faults are detected.

Remark 1. The design of Q for the parametric/uncertain case relies in the identification of the Dual Youla while the design for the additive faults is independent of it. Both branches of the algorithm re-design the controller component iteratively to address the new demands on robustness while optimizing the filter performance as a result of the integrated framework.

Note that the Q-parameter design stage could be based on a selection of off-line designs which are plug in the new controller using the Youla LFT representation.

An important assumption is that the designed controller provides an adequate level of robustness (together with a necessary safety factor but not a worst-case robust design in any instance). This assumption is widely used in the adaptive control literature although from a practical point of view it is critical and still an open research question.

Remark 2. Notice in step 1.b that the uncertainty estimate $\hat{S}_1$ can be rewritten as follows:

$$\hat{S}_1 = \tilde{M}_o \left(G_1 - G_o\right) M_o(\hat{S}_1) = \tilde{M}_o \left(G_1 - G_o\right) M_1 \qquad (36)$$

where the double Bezout identities: $(V_o\tilde{M}_o - N_o\tilde{U} = I)$ and $(-M_o\tilde{U}_o + U_o\tilde{M} = 0)$ are used in (30) to get:

$$\begin{aligned} G_1 &= \left(N_o + V_o\hat{S}_1\right)\left(M_o + U_o\hat{S}_1\right)^{-1} \\ &= \left(N_o + (I + N_o\tilde{U})\tilde{M}^{-1}\hat{S}_1\right)\Delta_{G_1} \\ &= \left(N_o\left(I + M_o^{-1}U_o\hat{S}_1\right) + \tilde{M}^{-1}\hat{S}_1\right)\Delta_{G_1} \\ &= N_o M_o^{-1} + \tilde{M}^{-1}\hat{S}_1 M_1^{-1} \end{aligned} \qquad (37)$$

$$\Delta_{G_1} = \left(I + M_o^{-1}U_o\hat{S}_1\right)^{-1} M_o^{-1} \qquad (38)$$

Hence, the estimate $\hat{S}_1$ is actually a frequency-weighted version of the difference between the nominal plants G_o and G_1, as indicated in [15] and in Section II.

V. Conclusions

In this paper an extension of the integrated (control and diagnostic) approach to the concept of nested structures is given. This extension hinges on the formulation of the integrated controller and uncertain plant as linear fractional transformations whose coefficient matrices cancel out. An algorithm for the nested integrated approach with the potential to detect parametric and additive faults has been given as well.

VI. Acknowledgements

This research was supported under NASA Langley Cooperative Agreement No. NCC-1-337, technical contract monitor Dr.Christine Belcastro.

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