NEW STRATEGIES FOR FAULT TOLERANT CONTROL AND FAULT DIAGNOSTIC

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Abstract: A recent architecture based on the Youla parameterization is analyzed for the nominal and uncertain case. This special architecture is characterized by a separation principle for the controller. In this paper, several stability results that allow to extend the structure to the integrated control/filter approach are presented. Two algorithms based on the recursive design and nesting of the integrated controller are also outlined.

Keywords: Fault detection and isolation (FDI), fault tolerant control (FTC), Youla parameterization, Dual Youla, nested structures.

1. INTRODUCTION

In the last thirty years the field of fault detection and isolation (FDI) and the associated fault tolerant control (FTC) have attracted much attention from control engineers, especially in the flight control community. Well-known ideas and methods from the control and the signal processing field have permeated from the beginning the FDI and FTC field. An example is the observer theory which underlays most of the developments in fault tolerant control and fault estimation, (Frank, P.M. and Ding, X., 1997; Chen, J. and Patton, R.J., 1999).

Generally, the control designer is only concerned with the design of a single robust controller where the only trade-off is the robustness (to disturbances and model uncertainties) versus the performance for the closed loop. More advanced techniques based on either a bank of controllers, reconfigurable or adaptive schemes have been proposed to improve the robustness/performance of the controllers. These controller often change as a function of scheduling parameters indicative of variations of the plant throughout the operational envelope. In this mind set, faults are considered as specially severe disturbances and worst-case design and analysis (e.g. $\mathcal{H}_\infty$ optimization or μ-synthesis techniques) are performed to evaluate the level of fault rejection. With the advent of the FDI field and the possibility of accurately identifying faults on the system this new information is incorporated to the design of controllers resulting in fault tolerant and reliable controllers.

Together with this potential to address fault scenarios a new trade-off arises between the controller and the FDI filter objectives (Wu, N.E., 1997). This trade-off has been analyzed in references (Nett, C.N. *et al.*, 1988; Stoustrup, J. *et al.*, 1997) where it was shown that the conflict stems from coupling of the objectives in the robust case (i.e. considering model uncertainty). Trying to address this trade-off a number of approaches have been proposed in the literature. In reference (Nett, C.N. *et al.*, 1988) an integrated control/filter approach was proposed and the opposing objectives qualitative analyzed. This integrated approach was later further developed in (Stoustrup, J. *et al.*, 1997) which proposed solutions in terms of the standard robust control configuration (Zhou, K. *et al.*, 1996). In reference (Wu, N.E., 1997), a mixed $\mathcal{H}_2/\mathcal{H}_\infty$ criterion was used to design a reconfigurable control with optimized control and diagnostic performance indexes. The Youla and the Dual Youla parameterizations were studied for high performance controllers in reference (Tay, T.T. *et al.*, 1998). A main result from this reference is that there is a separation principle for the controller. This separation principle and its connections with the residual generation theory have been used recently in a breadth of papers proposing different architectures for the controller and the diagnostic filter design, see (Suzuki, T. and Tomizuka, M., 1999; Stoustrup, J. and Niemann, H., 2001; Zhou, K. and Ren, Z., 2001). These developments have opened a new direction on the implementation and design of fault tolerant controllers. The explicit dependency of these novel architectures on the residual generator theory can further be exploited to tackle the trade-off that exist between the controller and the filter.

The main goal of this paper is to study these recent architectures from the perspective of the integrated control/filter approach and to extend the results to nested structures. Based on this study two algorithms are outlined for the solution of the residual generation fault tolerant control.

2. AN INTEGRATED FAULT TOLERANT ARCHITECTURE

In this section several theoretical concepts related to the parameterizations of all stabilizing controllers and all generators for a given plant are reviewed. It is shown how these two parameterizations can be combined in an integrated controller/filter structure based on a variant architecture of the well-known Youla parameterization. This architecture is analyzed for the nominal and the uncertain cases. Internal stability results for the integrated case are presented.

The class of systems considered can be represented by the following input/output realization

$$y(s) = G_u(s) \cdot u(s) + G_f(s) \cdot f(s) + G_d(s) \cdot d(s) \qquad (1)$$

where the transfer function $G_u(s)$ determines the effects on the system of the known inputs, $u(s) \in \mathcal{R}^m$; $G_f(s)$ those from the faults, $f(s) \in \mathcal{R}^n$; and $G_u(s)$ the effects of the disturbances, $d(s) \in \mathcal{R}^l$. Assume $G_u(s), G_f(s), G_d(s)$ are known and that no uncertainty is present. Notice that this model considers additive faults (i.e. faults modeled as disturbances) which affect the performance but not the stability of the system.

Using the conventional coprime factorization approach it is possible to parameterize all stabilizing controllers for a given system in terms of a free stable parameter $Q_c(s) \in \mathcal{RH}_\infty$, see for example (Zhou, K. *et al.*, 1996; Tay, T.T. *et al.*, 1998). Let the system $G_u(s)$ and a stabilizing controller, $K_o(s)$, have the following right and left coprime factorizations

$$G_u = N_u \, M^{-1} = \tilde{M}^{-1} \tilde{N}_u \qquad (2)$$

$$K_o = U \, V^{-1} = \tilde{V}^{-1} \tilde{U} \qquad (3)$$

where $N_u, M, \tilde{N}_u, \tilde{M}, U, V, \tilde{U}, \tilde{V} \in \mathcal{RH}_\infty$ must satisfy the double coprime factorization

$$\begin{bmatrix} \tilde{V} & -\tilde{U} \\ -N_u & M \end{bmatrix} \begin{bmatrix} M & U \\ N_u & V \end{bmatrix} = \begin{bmatrix} M & U \\ N_u & V \end{bmatrix} \begin{bmatrix} \tilde{V} & -\tilde{U} \\ -N_u & M \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} \qquad (4)$$

The class of all proper controllers stabilizing the system (1) is given by

$$\begin{aligned} K(Q_c) &= (\tilde{V} + Q_c \tilde{N}_u)^{-1} (\tilde{U} + Q_c \tilde{M}) \\ &= K_o + \tilde{V}^{-1} Q_c (I + V^{-1} N_u Q_c)^{-1} V^{-1} \end{aligned} \qquad (5)$$

for $Q_c(s) \in \mathcal{RH}_\infty$, such that $det(I + Q_c \tilde{N}_u \tilde{V}^{-1})(\infty) \neq 0$ An equivalent definition exists using a right factored form, see (Tay, T.T. *et al.*, 1998).

The controller given in equation (5) can be realized using a lower linear fractional transformation (LFT), $K(Q_c) = F_l(J_{nom}, Q_c)$, see also Figure 1,

$$F_l(J_{nom}, Q_c) = J_{11} + J_{12} Q_c (I - J_{22} Q_c)^{-1} J_{21} \qquad (6)$$

where the coefficient matrix, J_{nom}, is given by

$$J_{nom} = \begin{bmatrix} U V^{-1} & \tilde{V}^{-1} \\ V^{-1} & -V^{-1} N_u \end{bmatrix} = \begin{bmatrix} \tilde{V}^{-1} \tilde{U} & \tilde{V}^{-1} \\ V^{-1} & -V^{-1} N_u \end{bmatrix} \qquad (7)$$

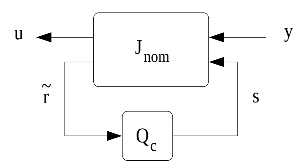

Fig. 1. Class of stabilizing controllers (lower LFT).

In (Tay, T.T. *et al.*, 1998) an architecture for high performance controllers is given, see Figure 2. This architecture is based on a modification of the Youla parameterization which allows for a separation principle on the controller and has connections with the generalized internal model control (GIMC) and the residual generation theory (RG), see (Zhou, K. and Ren, Z., 2001).

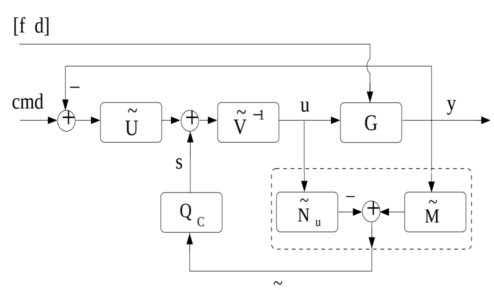

Fig. 2. Fault Tolerant GIMC-Youla Structure.

The Youla parameter $Q_c(s)$ can be interpreted as a robust controller that only takes action in the feedback loop whenever exogenous signals (e.g. disturbances, faults) or model uncertainties are present, i.e the primary residual, $\tilde{r}(s)$, is non-zero. An equivalent LFT can be obtained for this architecture, see Figure 3

where the coefficient matrix, $J_{FT-GIMC}$, is given by

$$J_{FT-GIMC} = \begin{bmatrix} \tilde{V}^{-1} \tilde{U} & -\tilde{V}^{-1} \tilde{U} & \tilde{V}^{-1} \\ V^{-1} & \tilde{N}_u \tilde{V}^{-1} \tilde{U} & -V^{-1} N_u \end{bmatrix} \qquad (8)$$

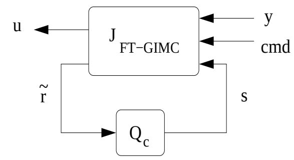

Fig. 3. Fault Tolerant - GIMC (lower LFT).

It is important to note that the effects of additive faults and disturbances do not affect the structure of the coefficient matrix. These effects influence the controller through the plant output channel, $y(s)$. Further, it is easy to show that the primary residual is given by

$$\tilde{r} = V^{-1}y + \tilde{N}_u\tilde{V}^{-1}cmd - V^{-1}N_u\, s = \tilde{N}_f\, f + \tilde{N}_d d \quad (9)$$

Thus, the primary residual is shown to be affected only by the additive faults and disturbances. Also note, that in the case the command inputs, $cmd(s)$, are zero the nominal coefficient matrix, J_{nom}, is recuperated.

The connection between the FT-GIMC structure and residual generation theory is easily shown using the following general result. The parameterization of all residual generators for system (1) can be given in terms of an additional free parameter $Q_f(s) \in \mathcal{RH}_\infty$, see (Ding, X. and Frank, P.M., 1990). The residual vector, r, can be generated using the following frequency domain residual generator

$$\begin{aligned} r = \; & Q_f{\cdot}\tilde{r} = Q_f\;(\tilde{M}{\cdot}y - \tilde{N}_u{\cdot}u) \\ & = Q_f\;(\tilde{N}_f{\cdot}\, f + \tilde{N}_d{\cdot}d) \end{aligned} \quad (10)$$

Note that the residual generator basically cancels the effect of the inputs of the monitored system in the nominal case. The different FDI approaches are basically based on the diverse ways available to select the transfer functions $Q_f(s), N_f(s), N_d(s)$ such that conditions on detectability, isolability, identification of faults and filter performance are fulfilled, see reference (Chen, J. and Patton, R.J., 1999).

The FT-GIMC structure can be converted into a Residual Generation - Generalized Internal Model Control (RG-GIMC) paradigm by incorporating the residual generation transfer function $Q_f(s)$, see Figure 4. It is important to highlight that this structure will be used for design purposes. The implementation will be similar but containing a supervisory module in charge of evaluating the residual and activating the fault tolerant controller as needed.

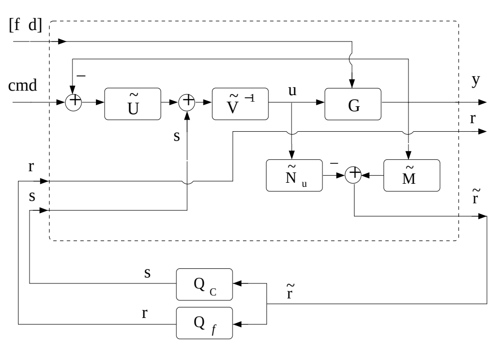

Fig. 4. Standard RG-GIMC paradigm.

The associated lower LFT representation for the RG-GIMC, $F_l(J_{RG-GIMC}, Q)$, is given by

$$\begin{aligned} \begin{bmatrix} \tilde{u} \\ \tilde{r} \end{bmatrix} &= J_{RG-GIMC} \begin{bmatrix} y \\ cmd \\ \tilde{s} \end{bmatrix} = \\ &= \left[\begin{bmatrix} \tilde{V}^{-1}\tilde{U} & -\tilde{V}^{-1}\tilde{U} \\ 0 & 0 \\ V^{-1} & \tilde{N}_u\tilde{V}^{-1}\tilde{U} \end{bmatrix} \begin{bmatrix} \tilde{V}^{-1} & 0 \\ 0 & I \\ -V^{-1}N_u & 0 \end{bmatrix} \right] \end{aligned} \quad (11)$$

where $Q = [Q_c \;\; Q_f]^\top \in \mathcal{RH}_\infty$, $\tilde{u} = [u \;\; r]^\top$ and $\tilde{s} = [s \;\; r]^\top$. Note that $J_{RG-GIMC}$ is exactly as $J_{FT-GIMC}$ augmented by the residual channel, $r(s)$.

The integrated (control/filter) controller, $\tilde{u}(s) = K_T(Q)\, y(s)$, can be parameterized as follows

$$K_T(Q) = \begin{bmatrix} K_o + \tilde{V}^{-1}Q_c(I + V^{-1}N_uQ_c)^{-1}V^{-1} \\ Q_f(I + V^{-1}N_uQ_c)^{-1}V^{-1} \end{bmatrix} \quad (12)$$

If the plant is stable (i.e. $\tilde{V} = V = I; \tilde{U} = U = 0; \tilde{N}_u = N_u = G_u; \tilde{M} = M = I$) the integrated controller given in reference (Stoustrup, J. *et al.*, 1997) is obtained.

The system given in equation (1) will have fault tolerant and fault diagnosis capabilities given appropriate stabilizing controller, $K_o(s)$, and coprime factorizations, equations (2-3), if stable, proper and real-rational matrices, $Q_c(s)$ and $Q_f(s)$, can be found such that the plant outputs, $y(s)$, behave as desired in the face of exogenous signals, $[f(s)\; d(s)]$, and that the residual, $r(s)$, identifies these exogenous effects. It is assumed from now on in the sequel that the commands are zero.

The corresponding closed loop assuming a generalized plant model, $\tilde{P}(s)$, is given in Figure 5. The exogenous input vector is given by $w(s) = [\; f(s) \;\; d(s) \;]^\top$, and the error/performance output by $e(s) = [\; e_c(s) \;\; e_f(s) \;]^\top$. The filter performance channel can be defined in a model-matching format, i.e. $e_f(s) = \mathcal{V}(s)f(s) - r(s)$. The transfer function $\mathcal{V}(s)$ is used to define the different FDI problems: e.g. for fault detection it can be a scalar, while for fault isolation a diagonal matrix might be more appropriate.

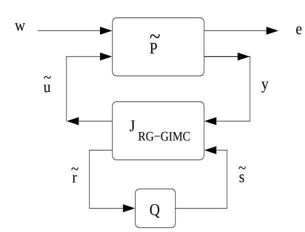

Fig. 5. Closed loop triad $(\tilde{P}, J_{RG-GIMC}, Q)$ $[cmd = 0]$.

It is straight forward to show that when $K(Q_c)$ stabilizes G then it can be implemented as in Figure 5 (using J_{nom} in that case) without loss of stability, see for example Chapter 2 in reference (Tay, T.T. *et al.*, 1998). This result carry over to the integrated case, $K_T(Q) = [\, K_1(Q_c)\; K_2(Q_f) \,]^\top$ seamlessly.

***Lemma* 1.** Let $(G, K_T(Q))$ be a stabilizing plant-controller pair with coprime factorizations given by equations (2-12). Then, the controller $K_T(Q)$ can be implemented as a lower LFT, $F_l(J_{RG-GIMC}, Q)$ with $Q = [\,Q_c \;\; Q_f\,]^\top \in \mathcal{RH}_\infty$ without loss of stability.

Figure 5 can be better represented using the standard feedback configuration, see (Zhou, K. *et al.*, 1996), as in Figure 6. where the generalized plant $\tilde{G}$ is given by

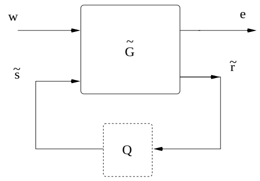

Fig. 6. Standard feedback (control/filter) paradigm.

equation (13)

$$\tilde{G} = \begin{bmatrix} \begin{bmatrix} G^*\tilde{N}_f & G^*\tilde{N}_d \\ \mathcal{V} & 0 \end{bmatrix} & \begin{bmatrix} G^*\tilde{N}_u\tilde{V}^{-1} & 0 \\ 0 & -I \end{bmatrix} \\ \begin{bmatrix} \tilde{N}_f & \tilde{N}_d \end{bmatrix} & \begin{bmatrix} 0 & 0 \end{bmatrix} \end{bmatrix} \quad (13)$$

$$G^* = (I + G_u K_o)^{-1}\tilde{M}^{-1}$$

Note that as expected $\tilde{G}_{22} = 0$, hence the closed-loop transfer function from exogenous inputs, $w(s)$, to error/performance outputs, $e(s)$, is affine in the parameter Q. This affineness on Q can be used to exploit numerical optimization techniques, (Tay, T.T. *et al.*, 1998). Standard approaches for the integrated design, (Stoustrup, J. *et al.*, 1997), can also be used using this set-up.

The Uncertain Case

In this section the uncertain case is analyzed. The uncertainty can be used to characterize modeling errors, parametric and/or structural faults, see (Jones, H.L., 1973; Niemann, H and Stoustrup, J., 2002). Parametric faults are usually associated with physical system parameters and are sometimes considered a more natural phenomenon, they can also de-stabilize the system.

The Dual Youla parameterization characterizes the class of plants stabilized by a given controller. Assume the controller is given by $K(Q) = U_Q\, V_Q^{-1} = \tilde{V}_Q^{-1}\tilde{U}_Q$ with $Q \in \mathcal{RH}_\infty$. The class of all proper linear plants stabilizable by $K(Q)$ is given by

$$\begin{aligned} G(S) &= (N_u + V_Q S)(M + U_Q S)^{-1} \\ &= N_u M^{-1} + \tilde{M}^{-1} S (I + M^{-1} U_Q S)^{-1} M^{-1} \end{aligned} \quad (14)$$

where $S \in \mathcal{RH}_\infty$ represents the Dual Youla parameter. Similarly as before, an (upper) LFT can be used to represent this family of plants.

where the coefficient matrix, $J_{S_{nom}}$, is given by

$$J_{S_{nom}} = \begin{bmatrix} -M^{-1}U_Q & M^{-1} \\ \tilde{M}^{-1} & N_u M^{-1} \end{bmatrix} = \begin{bmatrix} -\tilde{U}_Q\tilde{M}^{-1} & M^{-1} \\ \tilde{M}^{-1} & \tilde{M}^{-1}\tilde{N}_u \end{bmatrix} \quad (15)$$

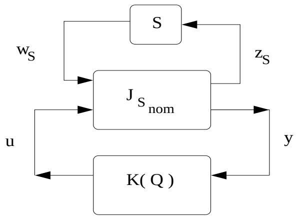

Fig. 7. Class of plants stabilized by $K(Q)$.

It is revealing to represent S in terms of the nominal plant, $G = NM^{-1}$, and the family of plants, $G(S) = N(S)M(S)^{-1}$.

$$S = \tilde{M}(G(S) - G)M(S) = \tilde{M}(S)(G(S) - G)M \quad (16)$$

An interpretation of S can be given as a frequency weighted measure of the difference between the nominal plant and the actual plant. Hence, any identification method of S can potentially lead to the detection of parametric faults, for an example see (Niemann, H and Stoustrup, J., 2002).

An important corollary is the possibility of representing an uncertainty model, S_o, as an upper LFT based on an 'approximate' model, $\hat{S}_1$, and an additional uncertain model, S_1, which captures the difference between S_o and its estimate, $\hat{S}_1$. More generally, $S_{i-1} = \hat{S}_i(S_i) = F_u(J_i, S_i)$ for $i = 1 \ldots m$

$$S_{i-1} = (N_{S_i} + V_{Q_i} S_i)(M_{S_i} + U_{Q_i} S_i)^{-1} \quad (17)$$

where it has been assumed $Q_i = U_{Q_i} V_{Q_i}^{-1}$ stabilizes both S_{i-1} and $\hat{S}_i = N_{S_i} M_{S_i}^{-1}$, and $J_i(\hat{S}_i, Q_i)$ is defined as in equation (15) with appropriate transfer functions.

Nested Structures

An important connection between the Youla and Dual Youla parameterizations can be made by combining the lower LFT for $K(Q_c)$ and the upper LFT of $G(S)$. Using equations (7) and (15) it can be shown that both interconnections in Figure 8 are indeed equivalent for any $S, Q \in \mathcal{RH}_\infty$. As a result it can be proved that given a stabilizing plant-controller pair (G, K_o), with coprime factorizations given as in equations (2-3), the pair $(G(S), K(Q_c))$, from equations (14,5), is stable if and only if (S, Q_c) is stabilizing, see (Tay, T.T. *et al.*, 1998), pg76. This stability result can be extended

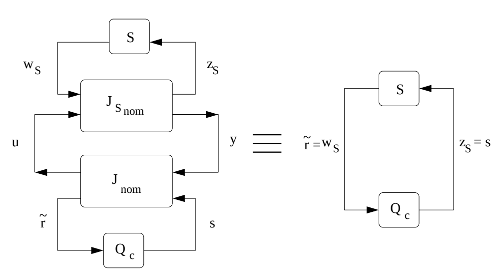

Fig. 8. Closed loop ($G(S), K(Q_c)$) $\equiv$ (S, Q_c).

to the integrated controller case as follows:

Theorem *1.* Let (G, K_o) be a stabilizing plant controller pair with coprime factorizations given as in equations (2-3). Let the pair $(G(S), K_T(Q))$ have the factorizations given in equations (14,12) where $K_T(Q)$ is the integrated controller and $Q = [\, Q_c \ \ Q_f \,]^\top$. Then, the pair $(G(S), K_T(Q))$ is stabilizing if and only if the pair (S, Q) is stabilizing.

The proof follows that of Theorem 4.4.2 in (Tay, T.T. *et al.*, 1998). It can be proved by arriving at the same diagram as in Figure 8 but using the coefficient matrices for $J^*_{S_{nom}}$ and $J_{RG-GIMC}$ (without the command channel) respectively. Define $J^*_{S_{nom}}$ as

$$\begin{bmatrix} z_S \\ y \end{bmatrix} = J^*_{S_{nom}} \begin{bmatrix} w_S \\ \tilde{u} \end{bmatrix} = \begin{bmatrix} -\tilde{U}\tilde{M}^{-1} & \begin{bmatrix} M^{-1} & 0 \end{bmatrix} \\ \tilde{M}^{-1} & \begin{bmatrix} \tilde{M}^{-1}\tilde{N}_u & 0 \end{bmatrix} \end{bmatrix} \quad (18)$$

combine with $J_{RG-GIMC}$. To achieve the desired result, remove the internal input/output signals to arrive at

$$J_{T_{11}} = -\tilde{U}\tilde{M}^{-1} + M^{-1}K_o\mathcal{J}\tilde{M}^{-1} \quad (19)$$
$$J_{T_{12}} = \begin{bmatrix} M^{-1}(K_o\mathcal{J}G_u\tilde{V}^{-1} + \tilde{V}^{-1}) & 0 \end{bmatrix} \quad (20)$$
$$J_{T_{21}} = V^{-1}\mathcal{J}\tilde{M}^{-1} \quad (21)$$
$$J_{T_{22}} = \begin{bmatrix} V^{-1}\mathcal{J}G_u\tilde{V}^{-1} - V^{-1}N_u & 0 \end{bmatrix} \quad (22)$$
$$\mathcal{J} = (I - G_uK_o)^{-1} \quad (23)$$

where the desired result is

$$\begin{bmatrix} z_S \\ \tilde{r} \end{bmatrix} = J_T \begin{bmatrix} w_S \\ \tilde{u} \end{bmatrix} = \begin{bmatrix} 0 & \begin{bmatrix} 1 & 0 \end{bmatrix} \\ 1 & \begin{bmatrix} 0 & 0 \end{bmatrix} \end{bmatrix} \quad (24)$$

Using the double Bezout factorization of equation (4) prove first $J_{T_{21}} = I$, then use it to solve the others.

In equations (14) and (17) it was shown that any plant can be characterized in terms of a parameter which on itself can also be parameterized in a recursive manner based on successive approximations. This indicates that any plant can be represented in an m recursive LFT whereby each successive design is a closer approximation to the real model.A similar result is valid for any controller where each approximation leads to the optimal controller. Theorem 1 can be extended to the generalized case and thus, the design of the controller can be simplified by designing the pair (S_i, Q_i) for $i = 0, 1 \ldots m$. The controller $K_i(Q_i)$ will stabilize both $G_o(S_o)$ and $G_{i-1}(S_i)$.

3. NESTED FAULT TOLERANT ALGORITHMS

In this section two possible algorithms based on nested structures are proposed. Both algorithms are based on the work related to nested (Q, S) design of (Tay, T.T. *et al.*, 1998). The first algorithm focuses in achieving the closed loop robust objectives under the abnormal situation. The second algorithm optimizes an off-line controller, selected to satisfy the new stability requirements, improving the closed loop and filter performance characteristics at each re-design. Both algorithms assume that the nominal plant, $G_o = N_oM_o^{-1}$, and a nominal stabilizing controller, $K_o = U_oV_o^{-1}$, are given.

An important assumption for the applicability of the algorithms is the availability of an identification scheme that estimates the uncertainty, e.g. (Hansen, F.R., 1989). As mentioned before, the identification of the Dual Youla can be used to identify parametric faults. The integrated residual generator could be used for this task in certain cases. An example of an FDI filter that estimates the parametric faults is given in (Felicio, P. *et al.*, 2002). For simplicity the residual part, Q_f, in the integrated Youla parameter is designed to optimize the detection and isolation of additive faults and severe disturbances, while the Dual Youla estimation scheme is used to identify parametric faults (uncertainty).

It is highlighted that the goal of the paper is not to address switching and design of controllers but to study the feasibility of using the RG-GIMC architecture and the results related to integrated nested structures to provide with alternative strategies for FTC.

1st Algorithm: "Robustness tuning"

The idea of the first algorithm is to design Q to improve the robustness of the closed loop in the face of faults (additive or parametric) and/or severe disturbances or uncertainty. At each re-design the performance of the closed loop and of the filter is reduced but otherwise is optimal for that design.

If parametric faults or critical uncertainty are detected:

1.a. Apply identification techniques to find estimate of the uncertainty model, $\hat{S}_1 = N_{S_1}M_{S_1}^{-1}$.
1.b. Find a robustifying integrated controller, $Q_1 = U_{Q_1}V_{Q_1}^{-1}$, that stabilizes $\hat{S}_1$.
1.c. Find the new estimated plant, $G_1 = G_o(\hat{S}_1)$, and new controller, $K_1(Q_1)$, using equations (14,12).

$$\begin{aligned} G_1 &= (N_o + V_o\hat{S}_1)(M_o + U_o\hat{S}_1)^{-1} \\ &= (N_oM_{S_1} + V_oN_{S_1})(M_oM_{S_1} + U_oN_{S_1})^{-1} \end{aligned} \quad (25)$$
$$\begin{aligned} K_1(Q_1) &= (U_o + M_oQ_1)(V_o + N_oQ_1)^{-1} \\ &= (V_oV_{Q_1} + M_oU_{Q_1})(V_oV_{Q_1} + N_oU_{Q_1})^{-1} \end{aligned} \quad (26)$$

1.d. Iterate if necessary starting from $G_1 = N_1M_1^{-1}$ and $K_1(Q_1) = U_1V_1^{-1}$ (i.e. find $\hat{S}_2$, Q_2, ...).

If additive faults are detected:

2.a. Find a robustifying integrated controller, $Q_1 = U_{Q_1}V_{Q_1}^{-1}$ that stabilizes the system.
2.b. Find the new controller, $K_1(Q_1)$, equation (26).
2.c. Repeat if new faults are detected.

Notice that the design of Q for the parametric/uncertain case relies in the identification of the Dual Youla while the design for the additive faults is independent of it. Both branches of the algorithm design the controllers to address the new demands on robustness while optimizing the filter performance through the integrated approach. The design stage could be as well a selection of off-line designs which are plug in the new controller using the Youla LFT representation. An important assumption is that the designed controller provides with the adequate level of robustness (plus a necessary safety factor if you will but it is not a worst-case design). This assumption is widely used in the literature although from a practical point of view it is critical and still an open question.

2nd Algorithm: "Performance tuning"

The idea of the second algorithm is to use a bank of (integrated) controllers designed off-line which are selected by a supervisory manager based on the information provided by the FDI filter and the identification scheme. After, the selected integrated controller is plug in the RG-GIMC architecture, further tuning can be performed to provide with the optimal (performance-wise) controller based on the Dual Youla identification scheme. If an additional condition appears, that warrants implementation of a new controller from the bank of filters, switching of the controller is performed and the performance tuning algorithm activated again. The advantage of this algorithm is that it has the potential to be used in conjunction with some of the available reconfiguration/switching schemes providing with a capability to improve the performance of the closed loop (and in the integrated case, the filter performance as well).

The algorithm is activated by the supervisory manager when indication of a fault, a severe disturbance and/or unacceptable model mismatch is identified. The supervisory manager selects the most appropriate (integrated) controller from the available off-line designs, $K(Q)_{1_{off}}$. It is probably not the optimal in terms of performance but adequate to robustly stabilize the plant under the new situation.

3i. Apply identification techniques to find estimate of the uncertainty model, $\hat{S}_1 = N_{S_1} M_{S_1}^{-1}$.
3ii. Find a performance-enhancer controller, $Q_1 = U_{Q_1} V_{Q_1}^{-1}$, that stabilizes $\hat{S}_1$. The design will be carried out to improve the performance of the closed loop and that of the filter. The same level of stability as $K(Q)_{1_{sel}}$ is expected.
3iii. Same as points 1.c-1.d in Algorithm 1.

4. CONCLUSION

In this paper a study of a recent parameterization architecture which favors a controller separation principle has been performed. The general stability results have been extended to the integrated control/filter approach and two FTC algorithms based on nested structures have been outlined.

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