# APPLICATION OF FDI TO A NONLINEAR BOEING-747 AIRCRAFT

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## Abstract

This paper presents a fault detection and isolation (FDI) filter design to linearized longitudinal dynamics of a Boeing 747 series 100/200. The FDI filter design based on fundamental problem of residual generation (FPRG). In our case the fault detection filter is sensitive to elevator and stabilizer fault. The three type of actuator failures considered in this paper are lock failure, loss in effectiveness and float failure. Typically, the FDI filter design is based on open loop linearized model and it is applied in the closed loop to a HIFI nonlinear model.

## 1 Introduction

Modern control systems are becoming more and more complex and control algorithms more and more sophisticated. Consequently, the issue of availability, reliability, operating safety are of major importance. These issues are important for safety critical systems such as nuclear reactors, cars and aircraft flight control systems. For safety critical systems, the consequence of faults can be extremely serious in terms of human mortality and environmental impact. Therefore, there is a growing need for on-line supervision and fault diagnosis to increase the reliability of such safety critical systems.

A traditional approach to fault diagnosis in the wider application context is based on hardware redundancy methods which use multiple sensors, actuators computers and software to measure and control a particular variable. In analytical redundancy schemes, the resulting difference generated from the consistency checking of different variables is called as a residual signal. The residual should be zero when the system is normal, and should diverge from zero when a fault occurs in the system. This zero and non-zero property of the residual is used to determine whether or not faults have occurred. Analytical redundancy makes use of a mathematical model and the goal is the determination of faults of a system from the comparison of available system measurements with a priori information represented by the mathematical model, through generation of residual quantities and their analysis.

There are various approaches to residual generation for, see *e.g.* the parity space approach [6], the multiple model method, detection filter design using geometric approach [11] or on frequency domain concepts [5], unknown input observer concept [3], dynamic inversion based detection [16].

The inversion based approach for LTI systems that can be used for detector design represented as minimum order stable linear system [16]. The outputs of these detectors are the failure signals while the inputs are the measured outputs and possible their derivatives. This makes not only the detection and isolation but also the estimation of the fault signals.

For the linear time invariant systems the problem of designing a stable filter capable of detecting the occurrence of a specific unmeasured input within a prescribe set is sometimes referred as the fundamental problem of residual generation (FPRG). In [12] it was shown the the existence of the solution of the FPRG depends on the relation between the subspace $\mathcal{L}$ determined by the direction of the failure to be detected and the minimal unobservability subspace containing the rest of the failure directions.

This paper is organized as follows. Section 2 gives a very quick review of the fundamental problem of residual generation. In section 3, the nonlinear and linear model for the longitudinal motion of the Boeing 747 is presented. The section 4 demonstrates the fault detection filter design based on geometric approach for Boeing 747 aircraft and the different actuator failure modes are considered. In this section the results of FDI filter simulation are presented for linear and non-linear model as well. The Section 5 consists of some concluding remarks.

## 2 Fundamental problem of residual generation

Let us consider the following LTI system, that has two failure events.

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + L_1 m_1(t) + L_2 m_2(t) \\ y(t) &= Cx(t) \end{aligned} \tag{1}$$

In equation (1), $x(t) \in \mathcal{X}$ is the state variable, $u(t) \in \mathcal{U}$ is the known control input, $y(t) \in \mathcal{Y}$ is the known output, the arbitrary time-varying functions $m_i(t) \in \mathcal{L}_i$ is the unknown failure modes. The term $L_1 m_1(t)$ represents the faulty behavior of the actuator that we are trying to monitor, i.e., a nonzero $m_1(t)$ should show up in the output of the residual generator $r(t)$. Similarly, $L_2 m_2(t)$ represents the faulty behavior of the other actuator which should not affect $r(t)$. As usual, our observables are the measurement $y(t)$ and the known actuation signal $u(t)$. The task to design a residual generator that is sensitive to $L_1$ and insensitive to $L_2$ is called the fundamental problem of residual generation (FPRG). [12]

Let us denote by $\mathcal{S}^*$ the smallest unobservability subspace (UOS) containing $\mathcal{L}_2$, where $\mathcal{L}_2 = \mathrm{Im} L_2$. $\mathcal{S}^*$ is the largest UOS in Ker$C$ containing $\mathcal{L}_2$. The $\mathcal{S}^*$ can be computed by *UOSA* algorithm [18]:

$$UOSA: \begin{cases} \mathcal{S}_0 = \mathcal{X} \\ \mathcal{S}_{k+1} = \mathcal{W}^* + (A^{-1}\mathcal{S}_k) \cap \mathrm{Ker} C \end{cases} \tag{2}$$

where $\mathcal{W}^*$ is the minimal (C,A)-invariant subspace containing $\mathcal{L}_2$. As it is well known, for LTI models, a subspace $\mathcal{W}$ is (C,A)-invariant if $A(\mathcal{W} \cap \mathrm{Ker} C) \subset \mathcal{W}$ that is equivalent with the existence of a matrix $D$ such that $(A + DC)\mathcal{W} \subset \mathcal{W}$.

**Proposition 1.** *FPRG has a solution if and only if* $\mathcal{S}^* \cap \mathcal{L}_1 = 0$, *moreover, if the problem has a solution, the dynamics of the residual generator can be assigned arbitrary.*

The equation $\mathcal{S}^* \cap \mathcal{L}_1 = 0$ indicates that $m_2(t)$ should not affects the output of the residual generator $r(t)$.

Given the residual generator in the form

$$\begin{aligned} \dot{w}(t) &= Nw(t) - Gy(t) + Fu(t) \\ r(t) &= Mw(t) - Hy(t) \end{aligned} \tag{3}$$

then $H$ is a solution of Ker$HC$ = Ker$C + \mathcal{S}^*$, and $M$ is a unique solution of $MP = HC$, where $P$ is the projection $P : \mathcal{X} \to \mathcal{X}/\mathcal{S}^*$.

In order to obtain the matrices in equation (3), consider $D_0$ such that $(A + D_0 C)\mathcal{S}^* \subset \mathcal{S}^*$, and denote by $A_0 = A + D_0 C|_{\mathcal{X}/\mathcal{S}^*}$. By construction, the pair $(M, A_0)$ is observable, hence there exists a $D_1$ such that the poles of $N = A_0 + D_1 M$ can be assigned arbitrary. Then set $G = PD_0 + D_1 H$ and $F = PB$.

Note that the important step in the design of the filter is to place the image of the second failure signature in the unobservable subspace of the residual $r(t)$.

Also the necessary condition simply states that the image of the first failure signature should not intersect the unobservable subspace of the residual generator, so that a failure of the first actuator show up in the residual $r(t)$.

Moreover, the failure signature $L_1$ is only used to check the solvability condition, and the actual construction of the filter is independent of $L_1$.

FPRG results can be extended to the case with multiple events. This has a solution if and only if $\mathcal{S}_i^* \cap \mathcal{L}_i = 0$, where $\mathcal{S}_i^*$ is the smallest unobservability subspace containing $\bar{\mathcal{L}}_i = \sum_{j \neq i} \mathcal{L}_j$. The block diagram of the extended FPRG (EFPRG) can be seen in Figure 1.

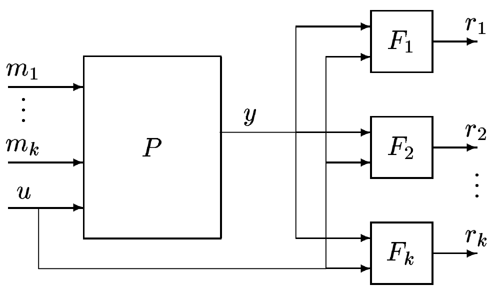


Figure 1: Block diagram of EFPRG

## 3 Longitudinal model of Boeing 747-100/200

The aircraft model used for FDI filter design is the Boeing 747 series 100/200. The Boeing 747-100/200 is an intercontinental wide-body transport with four fan jet engines designed to operate from international airports. The longitudinal control is performed through a movable horizontal stabilizer with four elevator segments (inboard and outboard elevators). Under normal operation the inboard and outboard elevators move together.

The nonlinear model for the Boeing 747-100/200 was obtained from reference [9, 10]. In reference [9] the longitudinal nonlinear model of the Boeing 747-100/200 is simplified by reducing the complexity of the aerodynamic coefficients while still maintaining a high degree of accuracy with respect to the full set of aerodynamic coefficients. The linear model obtained by Jacobian linearisation method is compared to nonlinear model. In order to validate the linear model around a certain operating point, closed loop time responses are obtained and then compared to the transient response of the nonlinear model. The Jacobian linearisation approach is the most widespread methodology to linearize nonlinear systems. It can be used to create linear model with respect to a equilibrium point that is included in the flight envelope of interest. The resulting model is an approximation to the dynamics of the nonlinear plant around that equilibrium point. Since it is a first order approximation it could lead to divergent behavior with respect to the nonlinear model for large control input. A detailed theoretical derivation of a Jacobian model for the Boeing 747-100/200 is given in [9].

The aircraft dynamics of B-747 are linearized for a straight level flight condition at 7000 m altitude and 241 m/s velocity (FPA = 0 deg). The linear model used for the FDI filter design is formed by augmenting the plant, obtained by linearizing the longitudinal dynamics of the aircraft at the above condition, with first order actuator dynamics and first order engine dynamics.

The linearized model can be expressed as follows.

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) + Du(t) \end{aligned} \tag{4}$$

The states are pitch rate $q$ (rad/s), total velocity $V$ (m/s), angle of attack $\alpha$ (rad), pitch angle $\theta$ (rad) as well as altitude $h_e$ (m). The measurements are fight path angle $\gamma$ (rad), acceleration normalized by gravity $\dot{V}/g$ (g), pitch angle $\theta$ (rad), pitch rate $q$ (rad/s), total velocity $V$ (m/s) and angle of attack $\alpha$ (rad). The inputs are elevon deflection $\delta_e$ (rad), throttle $T$ (N) and stabilizer defection $\delta_{st}$ (rad). The elevator actuator dynamics and the engine dynamics are first order transfer functions where the elevator is $G_{el} = \frac{37}{s+37}$ and the engine is $G_{eng} = \frac{0.5}{s+0.5}$.

In order to compare the linear model to the nonlinear model a 3 deg FPA command from 5 to 60 seconds is applied. The FPA command goes up to 3 deg from 5 sec and it goes to zero at 60 sec. The simulation time is 100 seconds. The comparison of the linear and nonlinear simulation can be seen in the Figure 2.

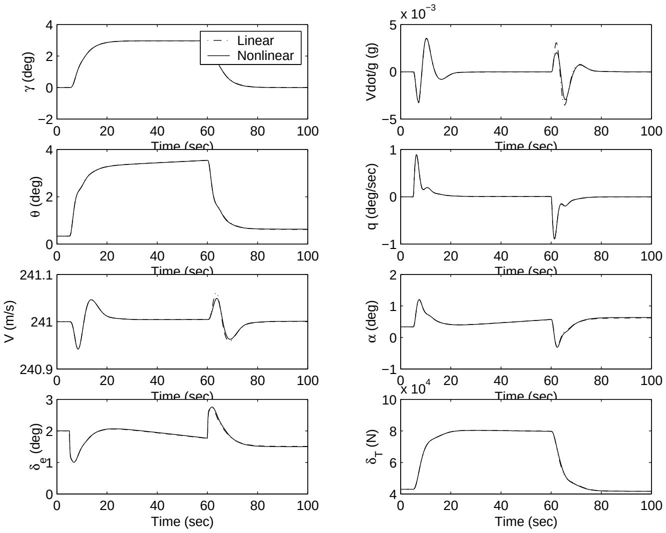


Figure 2: Linear model based on Jacobian method

The linear model based on Jacobian method and the nonlinear model time responses match each other almost perfectly. However, it is observed that the linear model is not able to follow the nonlinear response at 60 seconds with respect to $\dot{V}/g$ channel and $V$ channel as well. The reason of this phenomena is the fact that the aircraft trim conditions changed during climbing mode. Considering all channels in steady state, the outputs of linear and nonlinear model show same values.

## 4 FDI filter applied to linear and nonlinear closed loop

In this section the fault detection filter designed for open loop is studied in linear and nonlinear closed loop. The FDI problem is considered for open loop using EFPRG concept for fault in elevator and stabilizer respectively [12].

Our system can be described by the following linear time invariant model:

$$\dot{x}(t) = Ax(t) + Bu(t) + b_{el}\nu(t) + b_{st}\mu(t)$$
$$y(t) = Cx(t) \tag{5}$$

The elevator and stabilizer failures can be modeled as an additive term in the state equation where the failure signature $b_{el}$, $b_{st}$ are same as the first and second column of $B$ matrix, that represents the elevator and stabilizer direction respectively.

Now, apply geometric approach based on unobservability subspace concept to (5). The EFPRG problem is finding $N_i$, $G_i$, $F_i$, $M_i$, $H_i$ such that the following transfer matrix relationships hold:

$$\begin{bmatrix} u & \mu \end{bmatrix}^T \rightarrow r_1 = 0 \tag{6}$$
$$\nu \rightarrow r_1 \quad \text{left invertable} \tag{7}$$

and

$$\begin{bmatrix} u & \nu \end{bmatrix}^T \rightarrow r_2 = 0 \tag{8}$$
$$\mu \rightarrow r_2 \quad \text{left invertable} \tag{9}$$

The relation (6) indicates that $\mu$ and $u$ should not affect the output of the residual generator $r_1$. The condition (7) states that if $r_1 = 0$, then $\nu$ must be zero, i.e., if the elevator fails, then its effect should show up in the residual vector $r_1$. In case of stabilizer fault similar relationships can be stated.

In order to implement the FDI filter in closed loop, a $\mathcal{H}_\infty$ controller based on Total Energy Control System (*TECS*) concept is used for longitudinal model. The $\mathcal{H}_\infty$ control design ideas are taken from reference [14] and applied on the Boeing-747 LTI model selected for this paper. The *TECS* integrates all longitudinal altitude and speed control functions. The *TECS* achieves consistent decoupled maneuver control for all command modes and flight conditions. The $\mathcal{H}_\infty$ control design objectives are to achieve decoupled $\gamma$ and $V$ response of the aircraft, increase $\gamma$ bandwidth independent of engine dynamics and reject disturbances (wind gust, sensor noise). In case of closed loop, the filter takes the outputs of the aircraft and the controller outputs which is the elevator deflection $\delta_e$ and the throttle $T$.

The FDI filter is tested during an aircraft maneuver. We have a $\gamma$ command as a square wave, starting at 5 sec and ending at 40 sec. The simulation results of the FDI filter in case of linear closed loop can be seen in Figure 3.

An 1 deg square wave elevon failure is simulated occurring from 10 sec to 30 sec and a 1 deg step sta-

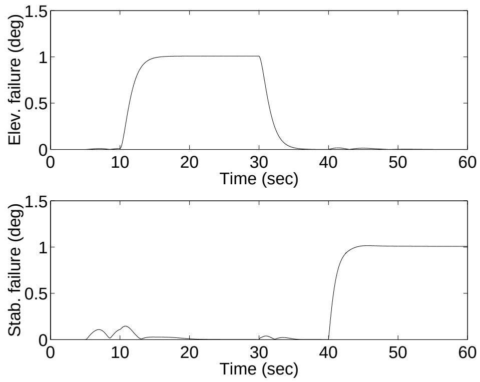


Figure 3: FDI filter applied to linear closed loop

bilizer failure occurring at 40 sec. The first residual shows the elevon fault and the second one is the stabilizer fault respectively. The effect of the two failures is decoupled and the residuals give an exact estimation of elevator fault and stabilizer fault. The impact of FPA command to residuals is negligible.

Next, the situation is studied when the FDI filter is applied to the nonlinear Boeing-747 simulation which represents the ”true” system. Figure 4 shows the simulation results of the FDI filter in case of closed loop nonlinear system.

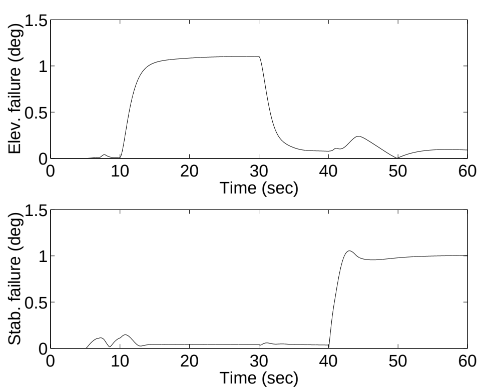


Figure 4: FDI filter applied to nonlinear closed loop

The fault scenarios are same as in previous case. The effect of the failures is decoupled and the residuals give an exact estimation of failures. In case of nonlinear closed loop there is a bump at 5 sec and at 40 sec in elevator fault residual and in stabilizer channel respectively, and a drift starting from 10 sec in stabilizer residual. The reason of this fact is that the FDI filter is analyzed using square wave $\gamma$ command. During climbing mode the trim conditions are changing that has effect at residual outputs. Moreover, the linear model is valid only near operating point where was linearized. The impact of FPA command in the nonlinear closed loop system appears but does not destroy the reliable operation of the detection filter if the nonlinear model remains near that operating point where the FDI filter is valid.

In practice, three fault scenarios can be taken into consideration for an elevator failure. One of them is the lock failure in elevator channel where the elevator remains locked at a particular position. In case of floating failure the elevator command changes instantaneously to $\alpha$. This means that the elevator cannot act against the wind to generate any force i.e. the elevator is floating. The third type of failure is the loss in effectiveness in the elevator channel. This means that the actuator effectiveness has reduced to some percentage. The simulation results of the FDI filter in case of elevator failures in linear closed loop can be seen in Figure 5. The Figure 5 consists of FDI filter outputs for all fault scenarios.

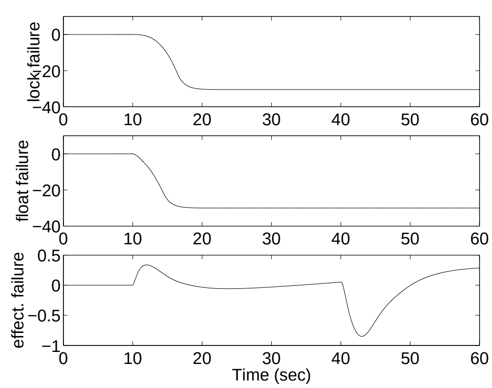


Figure 5: Simulation results in linear closed loop

In all cases the elevon failure occurs at 10 sec. At present, the controller does not reconfigure after the elevator failure occurs. Thus the stabilizer is not used at any instance by the controller.

In case of lock failure, the elevator gets locked at 1.5 deg at 10 sec. The first plot in Figure 5 shows the FDI filter output for lock failure. The FDI filter is able to detect the fault. When the lock failure occurs, the control action becomes more and more so that the controller compensates the aircraft motion besides failure. But the actuator does not work to generate any control force due to the fault, so the control signal goes to saturation. The FDI filter detects the appearance of failure, but it does not tell us the position where the elevator have locked. This is because the fault detection filter estimates the magnitude of the additive term $\nu$ in Equation 5. However, the position of elevator in case of lock failure comes from the sum of actual control action and $\nu$. Therefore, the actual control action has to be added to the residual output so that we get the position of elevator. In order to get the exact elevator position, the steady state gain of FDI filter has to be unit that is the filter has to capture exactly the magnitude of fault. The fault detection filter is able to detect if there is an actuator fault or not but a postprocess has to be used so that we tell the elevator position. The first step is the fault detection and the second step is called as fault identification.

The second plot in Figure 5 shows the simulation result in case of float failure. Now, the value of elevator command is equal with $\alpha$ after fault. The FDI filter is able to detect the fault. The behavior of controller is similar to lock failure from the control action point of view. The elevator command saturates because the actuator cannot generate any force against wind. It is very difficult to distinguish the lock and float failure because in both cases the actuator is not working after the fault and the residual signals is very close to saturation value. We need a postprocess to tell which fault has occurred. Adding the actual elevator command provided by controller to the residual output, in case of lock failure the difference is a constant signal while in case of float failure the difference is equal to angle of attack.

The third type of failure when the elevator actuator loss in effectiveness (third plot in Figure 5). In case of this failure the actuator is working when fault has occurred. Although, the actuator losses its effectiveness but the aircraft motion can be controlled with increased control action. The FDI filter detects the difference between the elevator command provided by controller and the effective control command. It is easy to tell this sort of failure from the previous ones because the shape of residual signal is similar to control signal.

The simulation results of FDI filter in nonlinear closed loop can be seen in Figure 6.

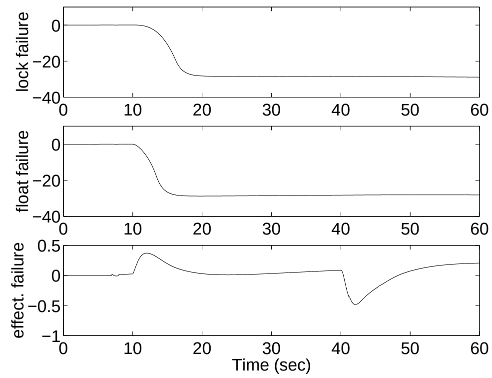


Figure 6: Simulation results in nonlinear closed loop

The outputs of FDI filter also shows good properties from detection point of view in nonlinear simulation. In case of lock failure as well as float failure there is no difference between the linear simulation and nonlinear one. The reason of this fact is that the actuator goes saturation in both cases and the effect of uncertainty caused by difference between linear and nonlinear model is negligible beside the saturation value of actuator. In case of third failure the shape of residuals are similar in both simulations but the magnitude of signals are different. In this case the actuator is working after the fault and doesn't rich the saturation value, therefore the effect of uncertainty is comparable to magnitude of residual. In case of effectiveness failure the FDI filter applied to nonlinear closed loop can detect the appearance of failure, but we don't get the exact estimation of degradation in effectiveness.

## 5 Conclusion

In this paper, a FDI filter design based on fundamental problem of residual generation concepts elaborated for LTI systems has been presented through the application of longitudinal model of Boeing 747-100/200. The FDI filter used in this paper is sensitive for elevator and stabilizer failure. The three type of actuator failures considered in this paper are lock failure, loss in effectiveness and float failure. The

FDI filter is designed for an open loop model of the system and then it is applied to the nonlinear closed loop system. The impact of FPA command in the nonlinear closed loop system appears but does not destroy the reliabile operation of the detection filter if the nonlinear model remains near that operating point where the FDI filter is valid.

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