Development of an Integrated LPV/LFT Framework: Modeling and Data-based Validation Tool

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***Abstract*— In this article the development of a software tool for LPV/LFT modeling and data-based validation techniques is described. This tool represents a step towards industrialization of these techniques, which although still suffering from a certain theoretical complexity are increasingly shown to help deal effectively with important real world applications. The tool is a key component for the development of an integrated LPV/LFT framework for control modeling, analysis and design which is also a necessity for successful transfer of LPV/LFT techniques to industry. The focus of the article is on describing the development of the integrated modeling/validation software and examining the applicability problems associated with these techniques. The technical readiness of the tool is exemplified through the development of validated LFT models used for LPV control design and analysis of a nonlinear longitudinal model of the NASA HL-20 re-entry vehicle.**

***Index Terms*— LPV/LFT modeling, LPV data-based validation**

I. Introduction

It is more than a decade ago that formal linear parameter varying (LPV) techniques [15], [35] made their appearance in the robust control literature as a natural extension of the popular robust linear time invariant (LTI) $\mathcal{H}_\infty$ system theory. The theoretical understanding and practical validity of LPV techniques is recognized by the numerous articles and industrial endeavors related to them [3], [21], [34], [41]. From a theoretical perspective, a control design LPV framework is already in the process of crystallization, in that most of the principal constituent elements have been fragmentally put into place: LPV system modelling [8], [26], [44], LPV system identification and validation [4], [39], [47], LPV performance and stability robustness theory [1], [18], [36], [52] and LPV control synthesis theory [24], [43]. Nevertheless, in order to ensure its successful transfer to industry adequate tools are needed for modeling, analysis and design [30]. From an industrial perspective, the modeling aspects are actually the key component in developing and using such an approach as they typically consume the most of the resources in a control development cycle. Indeed, part of the industry's reticence towards LPV synthesis and analysis techniques is the intrinsic difficulty of LPV modeling and the lack of validated tools to perform such a task in a methodological manner.

The main modeling representation used in modern control is the linear fractional transformation (LFT). This is so because the powerful modern robust analysis and synthesis techniques are generally based on LPV concepts and models that rely on representing the system in LFT form. LPV systems, and in full generality nonlinear systems given by polynomial or rational expressions, can be represented as LFT models. A LFT model can be simply described as a feedback connection between a bounded and unknown Δ operator – where the parameter-varying, uncertain or nonlinear terms are placed – and aknown operator M. In obtaining ”adequate” LPV/LFT models there are two main steps: the first is to obtain the model and the second to validate it.

For the first step, control designers generally use a family of LTI plants at different uncertainty values and points of interest throughout the operational envelope so that the parameter dependency is implicitly included. Subsequently, if a LPV model is required a fitting procedure is performed explicitly introducing the scheduling parameter dependency [2], [14]. Alternatively, if a parametric uncertainty description is required in LFT form starting from a state-space representation

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then strong physical understanding of the system the state-space represents is required so that multiplicative uncertainty models can be used to overbound specific coefficients of the LTI matrices (missing among other things parameters' relations, e.g. between mass and moments of inertia) prior to applying the LFT process of ”pulling out the deltas” [12], [53]. Main problems arising from such LPV/LFT modeling approaches are the high-level of conservatism and the high dimensional parameter blocks of the resulting LFT models, which might easily preclude the use of modern design and analysis algorithms. Additionally, such approaches necessitate of considerable time and effort. Solutions connecting both approaches have been proposed [25], [49] where numerical LTI models are obtained first and subsequently they are augmented with explicit symbolic parameter dependency so that in a final step, LFT-based parametric descriptions are automatically generated. The advantages of using symbolic processing techniques for LFT manipulation and order reduction has sparked the appearance of several approaches and toolboxes that systematize the LFT modeling process [6], [17], [25], [28], [37], [40].

Although the modeling step is an important precursor to system analysis and controller design, before this description can be used by the control engineer it must be validated. Model validation [23], [38], [45] provides a systematic way to evaluate the ability of a proposed model to represent observed system behaviors: i.e., given experimental data corrupted by additive noise find whether or not this data could have been produced by a combination of a nominal model and an uncertainty structure. Further, as the assessment of the quality of a model cannot be decoupled from the purpose for which the model is to be used the field of identification for control has received considerable attention [4], [11], [16], [48]. This field has focused on the design of identification criteria that delivered a control-oriented nominal model with most of the approaches addressing deterministic model validation in a robust control domain -thus, requiring a model representation in LFT form. The results show that in the case of unstructured uncertainty, model validation reduces to a convex optimization problem that can be efficiently solved [47], while allowing for additional structure can make the model validation problem difficult to solve requiring weaker conditions [13]. Although model validation techniques still suffer from a certain theoretical complexity they are increasingly shown to help deal effectively with the problems associated to real world applications [39].

As mentioned before, modeling and model-validation are both indispensable components of the ”modeling” step found in any control design cycle. Since LFT representations are found to be a nexus for both components it is then natural to rely on them to integrate the two components. In order to successfully transfer these techniques to an industrial setting, reliable and, to a certain extent, automated software tools are necessary.

This work is part of a European Space Agency (ESA) project where industrial and academia consortiums have been tasked with developing an industrial LPV control design framework supported with reliable software tools for its application to space systems. Indeed, the lack of such tools and framework had been identified by ESA as a key factor in the slow introduction of LPV techniques in space industry. The LPV modeling, analysis and design (LPVMAD) consortium lead by Deimos Space (Spain) is composed of research teams from the Computer and Automation Research Institute (Hungary), Delft Technical University (The Netherlands) and the University of Leicester (United Kingdom). A LPV framework developed around the standard control modeling, synthesis and analysis process has been put into place [30] together a set of toolboxes connecting these three main steps under the umbrella of LPV/LFT concepts. This paper focuses in the key element of modeling, and presents the development of an integrated LFT/LPV modeling and data-based validation software tool. An equally important contribution of this paper is the demonstration of the applicability of the modeling tool and of the resulting models for control and analysis purposes.

The layout of the paper is as follows: Section II provides a theoretical introduction to the main LPV/LFT modeling and data-based validation concepts and approaches. This section also includes an assessment of the difficulties associated with the applicability of these techniques to real, complex systems. Section III describes the development of the integrated LPV/LFT modeling and data-based validation software tool. Finally, Section IV serves as an assessment of the technological readiness level of the developed modeling tool by summarizing the control and analysis results obtained from using the developed validated models on a nonlinear simulation model of the NASA HL-20 re-entry vehicle.

II. Theoretical review of LPV/LFT modeling and data-based validation

A. LPV/LFT modeling

As previously mentioned, a LFT model (Figure 1) is a representation of a system using a feedback interconnection between two operators, a known, causal and stable

$M = [M_{11}\ M_{12};\ M_{21}\ M_{22}]$ and a causal, bounded Δ of proper dimension:

$$\mathcal{F}_L(M,\Delta) = M_{11} + M_{12}\Delta(I - \Delta M_{22})^{-1}M_{21} \quad (1)$$
$$\mathcal{F}_U(M,\Delta) = M_{22} + M_{21}\Delta(I - \Delta M_{11})^{-1}M_{12} \quad (2)$$

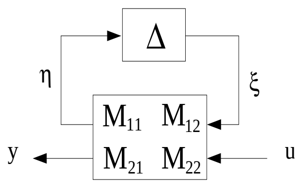

Fig. 1. Upper LFT graphical representation.

Δ is typically norm-bounded, $||\Delta||_\infty \leq 1$, but otherwise unrestricted in form (structured/un-structured) or type (nonlinear/time-varying/constant). If some of the components in the Δ operator are scheduling parameters a LPV system is obtained (if it contains states, the model is more appropriately referred to as quasi-LPV [26]). Indeed, a LFT model is just a structured representation of a LPV system. Two very important properties of LFT systems are that their interconnection (e.g. sum, concatenation) always results in another LFT [25], and that unstructured uncertainty at a lower level results in structured uncertainty at a higher level.

The order of a LFT is the dimension of the Δ matrix – for a diagonal uncertainty matrix $|\Delta|$ is the number of parameters, including repetitions, contained in the uncertainty operator, e.g., for $\Delta = diag(\delta_1 \times I_2, \delta_2) \rightarrow$ the LFT order is 3. Since many systems result in very high order LFT's if a realistic design or analysis problem is formulated, it is vital to have efficient and automated tools to find an almost minimal representation among all possible LFT representations of the system.

The most widespread approach in industry to obtain LFT models is based on: (i) the application of standard numerical Jacobian linearization to a computer implementation of the nonlinear system, at a sufficiently rich set of equilibrium points and uncertainty values, followed by (ii) polynominal fitting and (iii) LFT transformation [14], [25], [49]. This approach has the advantages of: relying on widespread numerical linearization techniques, requiring only standard (e.g. Simulink/Matlab) computer implementations of the nonlinear system, and resulting in purely LTI models once the components in Δ are assigned numerical values. These advantages are very relevant since most engineers are acquainted with the required numerical/computational modeling techniques and are also very knowledgeable on linear analysis/synthesis techniques, which are applicable to LFT models developed in this manner due to their underlying LTI nature. The drawbacks are a loss of modeling insight (when performing the numerical linearization) and the local nature of the LFT model with respect to the dynamics of the nonlinear system. Practical issues are the selection of the equilibrium points and the definition of the appropriate parameter-dependent polynomial fitting rule. Also, the approach might yield highly conservative and high dimensional LFT models, which might preclude the application of synthesis and analysis techniques.

The actual LFT modeling step is based on the so-called ”pulling out the deltas” method. This (non-unique) process obtains the appropriate M_{ij} components in Eq. 2 for a structured $\Delta = \{diag(\delta_1, \ldots \delta_U) : \delta \in \mathcal{R}\}$, where some of the δ_i might be equal:

1) Given a nonlinear system (typically its computer, e.g. Matlab/Simulink, implementation):

$$\dot{x} = f(x,u,\delta) \quad (3)$$
$$y = g(x,u,\delta) \quad (4)$$

where the state, input and output vectors are respectively x, u and y, and the δ vector represents uncertain or measurable scheduling parameters.

2) Perform (numerical) Jacobian linearization of the system at many operating conditions to obtain state-space matrices capturing the operational envelope:

$$\begin{bmatrix} \dot{x}_\sigma \\ y_\sigma \end{bmatrix} = \begin{bmatrix} \nabla_x f & \nabla_u f \\ \nabla_x g & \nabla_u g \end{bmatrix}_i \begin{bmatrix} x_\sigma \\ u_\sigma \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}_i \begin{bmatrix} x_\sigma \\ u_\sigma \end{bmatrix} \quad (5)$$

where the terms $x_\sigma(t) = x(t) - x_{eq}$ and $u_\sigma(t) = u(t) - u_{eq}$ represent deviation variables, while $\nabla_* f(x,\delta)$ indicates derivative of the function $f(x,\delta)$ with respect to the state or input vectors (the parameter vector δ is assumed constant here).

3) Represent the variation of the δ parameter vector for the complete family of LTI plants. This entails fitting a symbolic polynomial, dependent on δ, to the state-space matrices, their coefficients or the equilibrium manifold. For example in the matrix-fitting case: $\dot{x} = (A_0 + A_1\delta_1 + A_2\delta_2^2)x + Bu$.

4) ”Pull out the deltas” to get the LFT in Figure 1. That is, map $\dot{x} = (A_0 + A_1\delta_1 + A_2\delta_2^2)x + Bu$ into:

$$\xi = \Delta\ \eta \quad \text{with} \quad \Delta = diag(\delta_1, \delta_2 I_2) \quad (6)$$
$$\eta = M_{11}\ \xi + M_{12}\ [x\ u]^T \quad (7)$$
$$\dot{x} = M_{21}\ \xi + M_{22}\ [x\ u]^T \quad (8)$$

Example An example of the last step can be given using the acceleration equation $a_x = F_x/m$ assuming uncertain mass $m = m_0(1 + \nu_m\Delta_m)$, where $||\Delta_m|| < 1$ and ν_m represents the percentage variation with respect to the nominal mass m_0:

$$a_x = (1 + \nu_m \Delta_m)^{-1} \frac{1}{m_0} F_x$$

Which by looking at the LFT general equation:

$$a_x = M_{22} F_x + M_{21} \Delta (I - \Delta M_{11})^{-1} M_{12} F_x$$

yields that the LFT is given by $M_{11} = M_{21} = -\nu_m$ and $M_{12} = M_{22} = 1/m_0$.

The latter step is non-trivial and up to recently, with the appearance of automated LFT software tools [6], [17], [25], [28], it was limited to relatively simple systems due to the complexities of the algebraic manipulations. Despite the availability of these LFT toolboxes, obtaining LFT models of reasonable complexity (i.e. order) is still quite involved since most of these tools are based on algebraic manipulations that still require expert knowledge for their application to real, complex systems. It is precisely to address this issue that the LPV/LFT modeling tool component presented in Section III-A was developed.

B. *LPV/LFT data-based validation approaches*

Model validation provides a systematic way to evaluate the ability of a proposed model to represent observed system behaviors: given experimental data corrupted by additive noise find whether or not this data could have been produced by a combination of a nominal model and an uncertainty structure.

Assuming, without loss of generality, a general uncertainty model described by:

$$y = M_{21}\xi + M_{22}u + M_{23}w \tag{9}$$

$$\eta = M_{11}\xi + M_{12}u + M_{13}w \tag{10}$$

$$\xi = \Delta\eta, \tag{11}$$

where $M_{ij} = \mathcal{F}_L(P_{ij}, \Gamma)$ are LPV plants that depend on the scheduling variables Γ through a lower LFT equivalent to that in Eq. 1, see Figure 2. The nominal model M_{22} can be arbitrary with the disturbance $w \in \mathcal{L}_2$. The measured input u might correspond to the excitation input in an identification experiment. In the model validation problem, the output y and the scheduling variables Γ are considered to be finite sequences corresponding to measurements on a finite horizon of length T.

Since the proposed model validation of the LPV/LFT framework is based on the LTI approach let us summarize first the key ideas. Consider a LTI LFT model of

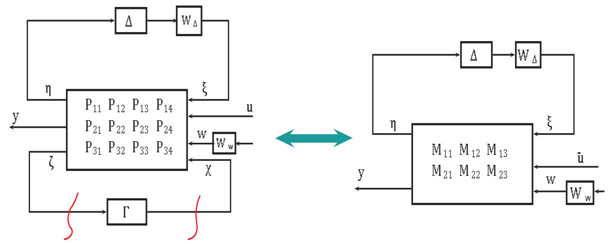

Fig. 2. Data-based model validation LPV/LFT framework

the form of the form Eq. 12-15. In order to validate the model, two issues have to be solved.

$$\dot{x} = Ax + B_\xi \xi + B_u u + B_w w \tag{12}$$

$$y = Cx + D_\xi \xi + D_u u + D_w w \tag{13}$$

$$\eta = Ex + F_\xi \xi + F_u u + F_w w \tag{14}$$

$$\xi = \Delta\eta, \tag{15}$$

First, it is necessary to formulate the problem in algebraic terms by expressing the signals w and (ξ, η) based on the measured/known values of the signals u and y. This task can be solved either in time domain (by assembling suitable Toeplitz matrices) or in frequency domain (relying on the corresponding transfer functions). Since the equations are overdetermined the solutions are parametrized using the corresponding kernels - denote by θ the corresponding parameters, for details see [22]. The time-domain approach results in a very large dimensional problem very difficult to solve and indeed the importance of setting the problem in the frequency domain is to be able to subdivide the problem into a series of smaller ones. Unfortunately, when constant parametric uncertainties are present, this is not possible directly – since the common constant values couple the individual simpler problems. This fact is often neglected in the literature. To cope with this problem these parameters can be considered as one dimensional full blocks. The values of the parameters should be estimated and introduced in the equations leading to a smaller problem that contain only the full blocks.

Having obtained this parametrized set of signals $(w(\theta), \xi(\theta), \eta(\theta))$, the second model validation step concentrates on the existence of a parameter θ_* such that there is a Δ_*, where $\xi(\theta_*) = \Delta_* \eta(\theta_*)$, with the norm bounds $||\Delta_*||_\infty \le \gamma$ and disturbance $||w(\theta_*)||_2 \le \gamma$ for a given $\gamma > 0$. Relying on tangential Carathèodory-Fejér interpolation results, either in time domain [46] or frequency domain [9], the existence of the desired Δ is guaranteed by conditions imposed on the norm of the signals $\xi(\theta_*)$ and $\eta(\theta_*)$ respectively.

Due to the structure imposed to the uncertainty Δ, finding an optimal uncertainty level γ that still allows a feasible solution θ_* is in general a nonconvex, NP hard task, which is often formulated as a set of bilinear matrix inequalities (BMI). The relation of the model validation problem with the skewed μ computation is revealed in [33]. Generally, to obtain numerically tractable problems some relaxation techniques are applied such as integral quadratic constraints (IQC) or the S-procedure (e.g., nonlinear functional approach), leading to a series of LMIs solved simultaneously with a bisection algorithm, see [10], [13].

In a LPV/LFT context the time domain approach can be directly applied, see [47]. However, for practical problems the size of the resulting systems is too big to be efficiently tractable. Therefore the proposed LPV data-based model validation procedure uses frequency-domain data and relies on the assumption that a basic LFT model structure has been already determined: defined by a nominal LPV plant model M with given scheduling variables, disturbance w and uncertainty model Δ. It then tries to answer if the LFT model passes the invalidation test by searching for sufficient invalidation conditions as these are the only conditions which lead to definitive statements about model quality.

The LPV/LFT model has the same structure as (12)-(14):

$$\dot{x} = Ax + B_\xi \xi + B_\chi \chi + B_u u + B_w w \tag{16}$$

$$\ldots$$

$$\zeta = Gx + H_\xi \xi + H_\chi \chi + H_u u + H_w w \tag{17}$$

$$\chi = \Gamma \zeta. \tag{18}$$

where χ and ζ corresponds to the scheduling variables Γ. The LPV problem is reduced to the LTI setting by considering χ as a known input variable and ignoring equations (17) and (18), resulting in the system formed by (12)-(14) being augmented only with χ as a fictitious input signal. It follows, that in order to apply frequency domain techniques the signal ζ must be available as a known input for the system. This implies that $H_w = 0$ and $H_\xi = 0$ must hold – which can be supposed without restriction – and that the term Gx should be known. This last condition represents the real restriction in the applicability of the method.

The uncertainty and disturbance weights W_Δ and W_w are usually introduced in the expression of the generalized plant. But since in a control oriented modeling and model validation framework their tuning plays an important role they are shown explicitly in Figure 2.

III. An integrated LPV/LFT modeling and data-based validation toolbox

The integrated LFT/LPV modeling and data-based validation software tool is developed to be used within a control design LPV framework [30]. It allows a control engineer to:

- From a modeling perspective, automatically obtain numerical LTI state-space plants (in LFT format) and to combine them into a LPV model (also in LFT form).
- From a validation perspective, compare the LFT-LTI and LFT-LPV plants to the nonlinear system using linear analysis approaches as well as LFT/LPV model validation approaches based on continuous-time data from the nonlinear system implementation.

A. LPV/LFT modeling tool

The LPVMAD modeling tool is a software toolbox composed of general Matlab/Simulink templates and specially defined functions supporting the LPV/LFT modeling of nonlinear and linear systems. The modeling toolbox interfaces with the user through a processing engine and relies, as primary inputs, on nonlinear and linear models or data.

Depending on the characteristics of the external model (nonlinear Simulink, nonlinear symbolic, LTI plant) there are three main LFT/LPV modeling algorithms available.

Algorithm 1

This algorithm represents the implementation of the exact nonlinear symbolic LFT approach proposed in [27]. It combines symbolic tools and automatic LFT generation approaches [25], [28] to exactly represent the complex ordinary differential equations (ODE) that define the nonlinear system of interest as a symbolic nonlinear LFT.

The main steps are the transformation of the nonlinear ODEs into a symbolic system where all the uncertain, nonlinear or unknown parameters are considered as symbolic δ's, and then "pulling out" these deltas (step 1 and 4 in Section II-A). It is remarked again that only until the recent appearance of LFT manipulation tools such as those in references [6], [17], [25], [28] it has been possible to perform complex system LFT derivations (and that the use of these tools still requires significant time in order to obtain a sensible LFT model).

The main differences of this LFT algorithm from those found in other toolboxes such as ONERA/DLR's LFR [17], [25] or that from NASA Langley [5] are the possibility to prioritize the reduction of specific uncertain parameters and its operation in a fully symbolic framework (allowing for symbolic terms in the operator M of the LFT). This latter capability can facilitate the search for general LFT models for specific problems, e.g., developing a general LFT for the longitudinal motion of a vehicle independently of the numerical values of the variables. The algorithm behavior is markedly different to that from the other toolboxes as was shown in [29].

Algorithm 2

It uses specialized LFT symbolic and Simulink tools to include parametric uncertainty in the standard Simulink implementation of a nonlinear system. Thus, when using numerical trimming and linearization routines a numerical LTI state-space is obtained directly in LFT form (i.e., specific input/output channels are obtained describing the feedback interconnection between the LTI nominal plant and the uncertainty structure). This algorithm defaults to the standard numerical LTI plants derivation when Δ is fixed at a specific value. Constant or purely additive uncertain parameters can not be tackled by the algorithm, although this limitation is applicable to any other LFT modeling if the uncertain parameter is not affine to an input or state, i.e. standing alone as in $y = x + \delta$.

For each vehicle, the user must create a dedicated directory and tailor the provided user-modifiable Matlab and Simulink templates. The resulting tailored files are then called by general purpose trimming, linearization, simulation and plotting M-scripts. The implemented tool guarantees harmonization and facilitates reduced system implementation and LFT/LPV modeling time. The main template file for this algoritm is a Simulink shell, see Figure 3, formed by four principal blocks: DKE (Dynamic-Kinematic-Environment), Sensor, Actuator and Uncertainty. The vehicle-specific shell is formed by appropriately changing specific sub-blocks within these four main blocks.

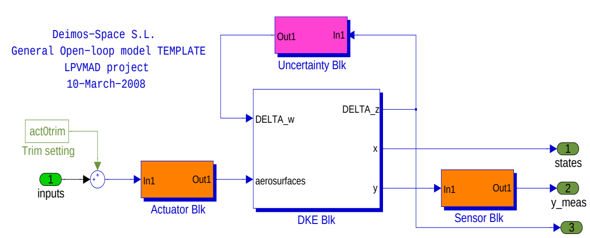

Fig. 3. Open-loop nonlinear main-layer Simulink template shell

The Simulink shell is characterized by using a LFT description that captures the introduction of the uncertainty in a physically meaningful manner. This LFT formulation is obtained by using algorithm 1 to obtain symbolic LFT representations which then can be introduced through M/C-code S-functions and/or directly transformed into Simulink blocks using LFT multiplicative uncertain models.

Example. The acceleration example from Section II-A can be used to exemplify the latter LFT Simulink implementation step. Starting from the LFT symbolic representation obtained in the previous example, based on a multiplicative uncertainty model and mapping the resulting equation into the standard LFT formulation from Eq. 2, a corresponding LFT Simulink block implementation can be given by Figure 4 where:

$$w_\Delta = \nu_m \Delta_m \, z_\Delta \tag{19}$$

$$z_\Delta = - w_\Delta + \frac{F_x}{m_o} \tag{20}$$

$$a_x = - w_\Delta + \frac{F_x}{m_o} \tag{21}$$

Note that the normalization gain ν_m and the nominal value m_o are not shown in Figure 4 since they are placed in a special uncertainty sub-block within the Simulink template that facilitates user-accessibility and provides modeling flexibility.

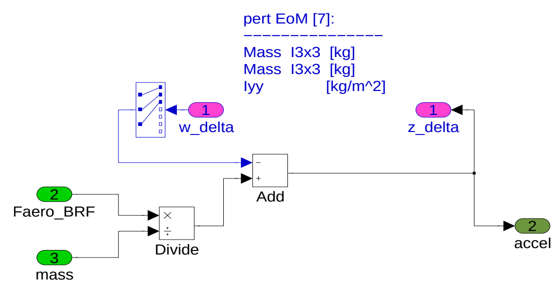

Fig. 4. Example Simulink implementation uncertain parameter

The additional channels (w_Δ, z_Δ), obtained by using the Simulink LFT implementation, represent the physical connection of the uncertainty with the system and allow linearizing with respect to perturbed uncertain variables. Thus, if the parametric Δ results in highly nonlinear behavior the resulting LTI-LFT system will be of necessity valid only for small uncertainty values. Nevertheless, the "physical" interconnection information absorbed in the resulting numerical LFT model is indeed the greatest advantage of the proposed method as it is expected that such a model will be less conservative (than a parametric uncertainty LFT obtained using the standard approach from a family of LTI models to a single LFT) and certainly easier to use for a wider group of design engineers (than current symbolic-base LFT methods).

It is noted that the presented approach differs to previous LPV/LFT modeling approaches [2], [14], [17],

[25] in that it combines symbolic LFT manipulations and standard (numerical) Matlab/Simulink implementation to automatically get numerical LTI-LFT plants. The method has parallels with that proposed in [7] in as much as that both attempt to use automated Simulink-based LFT modeling tools.

Algorithm 3

This last algorithm implements the more traditional form of LFT/LPV modeling, that of scheduling a family of LTI plants [2], [14]. It starts from a set of numerical LTI plants, and associated equilibrium set, and helps the user to explicitly introduce symbolic scheduling vectors and interpolation rules dependency. Once the matrices are symbolically scheduled, thus yielding a symbolic LPV system, it is straightforward to apply LFT manipulation algorithms [25], [28] to get a symbolic LFT-LPV system. The algorithm facilitates the inclusion of dynamic uncertainty (e.g., frequency-dependent models arising from neglected flexible modes) but can also be used to introduce parametric uncertainty (directly to the LTI state-space matrix coefficients). The symbolic fitting is performed through a least-square method and is amenable to further weighted decompositions [51].

B. LPV/LFT data-based validation tool

The primary function of the toolbox is to provide a computationally tractable method to (in)validate LFT/LPV models. In exactitude, the toolbox allows testing that the acquired measurement data does not contradict the given uncertainty structure and weights W_Δ, W_w associated with a nominal LPV plant. A secondary function is to facilitate a joint uncertainty modeling and controller synthesis process by simultaneously tuning W_Δ and W_w in the set of all admissible weighting functions that satisfy the model validation problem.

Time domain data algorithms

For time-domain (TD) data, the toolbox contains two routines: the first considers an unstructured LTI uncertainty set while the second is developed for both structured/unstructured, possible LTV, uncertainty sets. The implementation is based on the algorithms presented in [47] and [46] respectively.

It is noted that despite the dimensionality problem for these validation methods, they seem to be more fitted for the LPV/LFT class of systems. And furthermore, the difficulties encountered in the time-domain to frequency-domain transition on signal level reveals the importance of searching for numerically reliable TD model validation algorithms. This is a topic under on-going investigation in the research community and the reason why these, for the time being relatively impractical, algorithms are implemented in the modeling tool.

Frequency domain data algorithms

The frequency domain validation tools are based on a data structure as sketched in Figure 2. Consider the general problem with a structured uncertainty $\Delta = diag\{\Delta_f, \Delta_c, \Delta_r\}$ containing full complex blocks, repeated complex scalar and repeated real scalar blocks. Then, a nonlinear optimization problem arises characterized by one linear equality constraint and several inequality and nonlinear equality constraints. The linear equality constraint is that the nominal model error is recovered by the uncertainty signals ξ and w:

$$e = y - M_{22}(\bar{u}) = \begin{bmatrix} M_{21} & M_{23} \end{bmatrix} \begin{bmatrix} \xi \\ w \end{bmatrix} \tag{22}$$

The first criterion of consistency is:

$$rank\,[M_{21}\, M_{23}] = n_y, \tag{23}$$

where n_y is the number of outputs. The solution of the linear equality constraint is parametrized in the following form:

$$\begin{bmatrix} \xi \\ w \end{bmatrix} = \begin{bmatrix} a_\xi \\ a_w \end{bmatrix} + \begin{bmatrix} B_\xi \\ B_w \end{bmatrix} \theta \tag{24}$$

where θ is to satisfy all other constraints and minimize the bound γ. The input of the perturbation block can be also parametrized by θ using:

$$\eta = \begin{bmatrix} M_{11} & M_{13} \end{bmatrix} \begin{bmatrix} \xi \\ w \end{bmatrix} + M_{12}\bar{u} = a_\eta + B_\eta \theta. \tag{25}$$

Finally the following conditions can be formulated:

$$\min_{\gamma^2, \theta, \delta_c, \delta_r} \gamma^2 \tag{26}$$

subject to

$$\frac{\|\xi_i(\theta)\|^2}{\|\eta_i(\theta)\|^2} \leq \gamma^2, \quad i = 1, ..., \tau \tag{27}$$

$$\|w(\theta)\|^2 \leq \gamma^2 \tag{28}$$

$$|\delta_{ci}|^2 \leq \gamma^2, \quad i = 1, ..., \tau_c \tag{29}$$

$$|\delta_{ri}|^2 \leq \gamma^2, \quad i = 1, ..., \tau_r \tag{30}$$

$$\xi_c(\theta) = \Delta_c \eta_c(\theta) \tag{31}$$

$$\xi_r(\theta) = \Delta_r \eta_r(\theta) \tag{32}$$

Observe that almost all of the constraints of the minimization problem are IQCs.

Apart of the special case when $M_{11} = 0$ and $M_{13} = 0$, the validation problem and the task to select proper weights for the given uncertainty structure leads to a set of BMIs, which is in general an NP hard problem. However, there is growing interest in approaches that facilitate the choice of these weights in a reasonable computational time.

The toolbox implements the method of [9] enhanced by an adapted D scaling process to provide an approximate solution of the optimization problem. The critical point of the solution is to find feasible initial parameters, since the nonlinear programming can be very sensitive for the initial conditions.

To enhance the possibility of tuning the weights in the validation routines the user can explicitly set the ranges considered for the frequency values of the different weights. The toolbox also contains features to facilitate the manipulation of scaling factors for the uncertainties and disturbances during the validation experiments and finally to obtain the weighting functions W_Δ and W_w by fitting suitable transfer functions on the frequency domain data.

Nonlinear function algorithm

When applying the nonlinear functional approach, [13], the disturbance (noise) level enters linearly in the resulting LMIs. The key point exploited by the method is that if G is a quadratic functional in the variable π, then $H(\theta) = G(\pi(\theta))$ is a quadratic functional in θ; given the quadratic functionals H_j if there exist scalars $\tau_i \geq 0$ such that $H_0(\theta) + \tau_1 H_1(\theta) + \cdots + \tau_N H_N(\theta) \leq 0$ holds for all θ, then the model is invalidated.

Therefore, an efficient search can be done for its suitable value by maintaining fixed the other parameters. Applying the S-procedure only for the IQC defined by a given block and the IQC for the disturbance, a lower bound for the individual block norms at given frequencies can be computed by fixing the noise level and performing a search (with bisection). Since the S-procedure is in general not lossless, the obtained bounds provide only an informative view on the true weights, which might be larger. However, the method is relatively cheap in terms of computational time, and therefore the validation toolbox also provides an implementation of this method.

IV. Application to a re-entry vehicle

An assessment on the technological readiness level of the developed tool was performed within the LPVMAD project using a simplified nonlinear atmospheric re-entry vehicle. The process and results obtained serve to exemplify the applicability, shortcomings and advantages of the integrated LPV/LFT modeling and data-based validation tool. References [31], [50], [51] provide further details on the modeling, design and analysis aspects respectively.

A. The nonlinear re-entry system

The NASA HL-20 lifting-body vehicle was proposed as a substitute of the U.S. Space Shuttle Orbiter. Although finally de-commissioned, many research efforts in the experimental and computational fields were performed to develop a fairly detailed baseline aerodynamic database for the complete operational envelope of the vehicle [19], [20]. For the purposes of this work, an implementation of the NASA HL20 nonlinear system for the longitudinal motion in the low-atmospheric stage is developed based on the NASA reports [19], [20]:

$$\dot{V} = \frac{F_{wx}}{m} - \frac{\mu_E}{R^2} sin\gamma \tag{33}$$

$$\dot{\gamma} = \frac{-F_{wz}}{Vm} - \left[\frac{\mu_E}{R^2 V} + \frac{V}{R^2}\right] cos\gamma \tag{34}$$

$$\dot{\alpha} = q + \left[\frac{F_{wz}}{gm} + cos\gamma\right] \frac{\mu_E}{R^2 V} \tag{35}$$

$$\dot{q} = \frac{M_y}{I_{yy}} \tag{36}$$

$$\dot{R} = V sin\gamma, \tag{37}$$

where R denotes the distance from the centre of gravity of the vehicle to the centre of the Earth, V represents the ground speed, γ is the flight-path angle, α is the angle of attack, m is the aircraft mass, and the pitch rate q is given with respect to the body-reference-axis. Earth is assumed to be an ellipsoid with mass symmetry about the polar axis and gravitational constant μ_E ($g = \mu_E/R^2 \approx 9.81$ m/s^2). F_* and M_* are respectively the aerodynamic forces and moments. The aerodynamic database covers a wide range of performance variation: Mach number $M \in [4.0, 1.5]$, altitude $he \in [30, 12]$ km, angle of attack $\alpha \in [-2, 30]$ degrees and sideslip angle $\beta \in [0, 2]$ degrees. The aerodynamic coefficients are obtained from more than 60 look-up tables that condense the effects from the different stability derivatives, each formed in turn by a polynomial function in Mach, α, β and control surfaces. The aerodynamic surface configuration consist of upper left/right flaps (DUL and DUR), lower left/right flaps (DLL and DLR), wing left/right flaps (DEL and DER) and rudder (DR). A Mach-based control surface mix logic transforms "pilot/control" elevator δ_{ele}, speed-brake δ_{sbk} and rudder δ_{rud} commanded deflections into aerodynamic surfaces deflections. An auto-land trajectory, following typical approach profiles for landing-able re-entry vehicles, is defined based on Mach, altitude and flight-path angle γ profiles [20].

The complexity of the nonlinear model used, even in the longitudinal motion, can be assessed by looking at the pole scattering across Mach shown in Figure 5.

B. LFT model

The objective of the LPV/LFT modeling stage is to capture in a single LPV model, with a LFT structure,

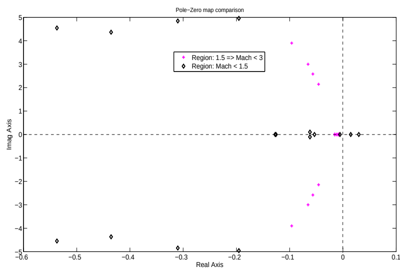

Fig. 5. Longitudinal motion HL20: pole/zero map across Mach

the dynamic behaviour of the HL-20 longitudinal motion along the defined trajectory. This LPV/LFT model will then be used for control design and analysis.

As typical for this type of systems the model is parameter-scheduled in Mach, $\Gamma = \{M \times I_n\}$, but also considers parametric Δ_{param} and aerodynamic uncertainty Δ_{aero}. Note that I_n indicates the number n of repetitions for each real parameter. In the present case, $\Delta_{param} = diag\{x_{cg} \times I_2, y_{cg} \times I_3, z_{cg} \times I_4, I_{yy} \times I_5, m \times I_6\}$ corresponding to center of gravity, moment of inertia and mass, and $\Delta_{aero} = diag\{C_L, C_D, C_M\}$ corresponding to the longitudinal aerodynamic coefficients of lift C_L, drag C_D and pitching moment C_M.

As it was mentioned before, the standard approach consists in obtaining a sufficiently rich set of LTI plants at different uncertainty values and flight conditions, to perform least square fittings to capture the variability in the scheduling and uncertain sets, and then to transform the system into a LFT. The process is quite time consuming, requires strong physical understanding of the system and typically results in LFT models of high dimension and conservatism.

For the HL20, the 2nd algorithm of Section III-A is used (after suitable tailoring of the templates to the present vehicle) to obtain four LTI/LFT models along the trajectory, selected at $M = [4.0,\ 2.75,\ 1.75,\ 1.5]$. Each plant has a dimension $29x23$ corresponding to 5 states, 7 outputs, 2 inputs and $16x16$ uncertainty channels –arising from the 5 parametric and 3 aerodynamic uncertainties. These 4 uncertain LFT state-space models are then symbolically least-square fitted on Mach and the resulting symbolic dependency ”pulled out” into the LFT as an additional scheduling block $\Delta_{sched} = \Gamma$ by using the 3rd modeling algorithms in Section III-A. The specific fitting approach [31] uses, coefficient by coefficient, symbolic 2nd and 3rd order polynomials in Mach to yield a LPV/LFT model with a scheduling parameter dimension of respectively I_{20} and I_{30} (plus an additional dimension of 16 from the Δ_{param} and Δ_{aero} blocks). Figure 6 graphically shows the process followed.

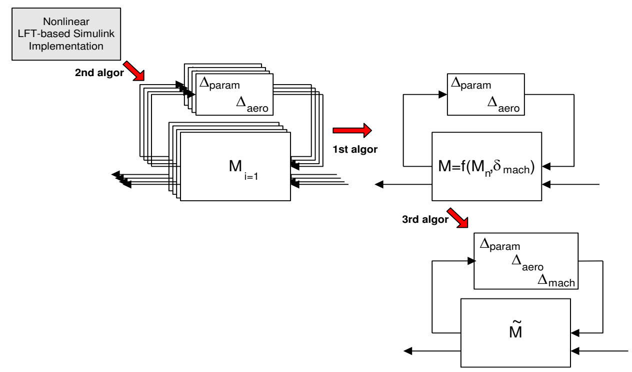

Fig. 6. LPV/LFT modeling tool process

As can be observed, the advantages of the developed tool are the direct physical incorporation of the (parametric and aerodynamic) uncertainty, its high degree of automatization, and its reliance on standard languages such as Matlab/Simulink which are wide spread in the modeling of nonlinear systems for control design and nonlinear simulation. These advantages serve to reduce the time-to-model while not significantly increasing the time-to-develop the Matlab/Simulink implementation of the nonlinear system, which is required in any case to perform nonlinear simulations and Monte Carlo studies for the final verification and validation stage.

Including the scheduling parameter dependency directly in the Simulink model is currently under investigation but it is noted that due to the local nature of the trimming and linearization processes the validity of such a LPV/LFT model will be of necessity highly problem dependent and likely limited (the same can be said of the uncertain LTI/LFT models if the uncertainty range induces complex nonlinear behavior).

The standard validation of the resulting *for-design* and *for-analysis* models is through comparison of the eigenvalues and frequency responses of frozen-time systems (at specific Mach and parametric uncertainties values) extracted from the LPV/LFT model with respect to the corresponding LTI systems obtained directly from the nonlinear system implementation. Figure 7 shows an example using the frequency domain responses [31]. It can be seen that both LPV/LFT models (the 2nd and the 3rd order Mach polynomial fitting) adequately capture the LTI dynamic characteristics.

Due to the smaller Mach-dimension of the 2nd order fit LPV/LFT model, this model is selected for the assessment of the LPV data-based validation component of

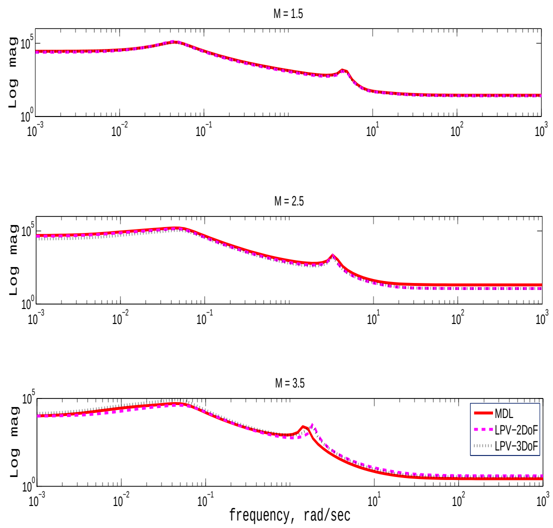

Fig. 7. LTI, LPV-2DoF fit & LPV-3DoF fit: frequency responses

the tool. For the LPV (IQC-based) control design, summarized in SubSection IV-D, a further simplified model is derived to facilitate quickly arriving to a preliminary control design which is subsequently matured using the 2nd order LPV/LFT, and the resulting controller finally analyzed using both 2nd and 3rd order fit LPV/LFT models in an incremental validation fashion.

C. Data-based validation

Additional validation tests are performed thanks to the data-based validation component of the developed modeling tool. The measured data for the nonlinear system comes from the Simulink implementation of the nonlinear model, which is considered as the 'real' system due to the impractical possibility of acquiring flight data for re-entry vehicles. It is noted that much emphasis is always given in space projects to develop nonlinear implementations that include all the main system and environmental features as well as the highest fidelity aerodynamic database (from wind tunnel and CFD experiments), which then become the baseline benchmark in the control design cycle.

Besides the verification of the tool, the aims of this assessment are: to investigate the sensitivity of the different data-based algorithms to the problem data, to investigate the conservatism introduced by their different theoretical assumptions, and to demonstrate the tool capability to find reduced-order weights that can explain the uncertainty present in the system. The latter aim is especially important for the future maturation and impact of the tool since calculation of validated *for-design* weights, given a LFT model and time/frequency domain data from the real system, will allow a designer reducing the iterative weight design process when using modern robust control synthesis approaches.

For the HL-20 LPV/LFT model, see [31] for specific details, several validation cases are tested assuming different values and structures for the perturbation w and the parametric uncertainty Δ. It is noted that the time-domain (TD) algorithms were not used due to the practical limitations in solving the resulting large-size validation problem. Also, it is important to remark that during the preparation of the data for the frequency domain (FD) algorithms, special care is always needed to transform the time-domain measurements into the frequency domain. In general, in order to obtain the FD signals zero initial conditions and final states are assumed. If these assumptions do not hold, the effect of their "side conditions" should be considered during the computations, i.e. a sufficiently good estimate of the initial and final state must be obtained. Numerical experiments show the importance of this correction.

To illustrate the output of a validation experiments, Figure 8 shows the validation result using the nonlinear functional approach for the case of a noise gain range of $\delta_d \in [0,\, 1]$ and a full-block uncertainty set with the same size as Δ_{param} and using a fixed $W_\Delta = 1$. It is seen that the model passes the (in)validation test for this case (a successful validation should yield a frequency response magnitude less than one across frequency).

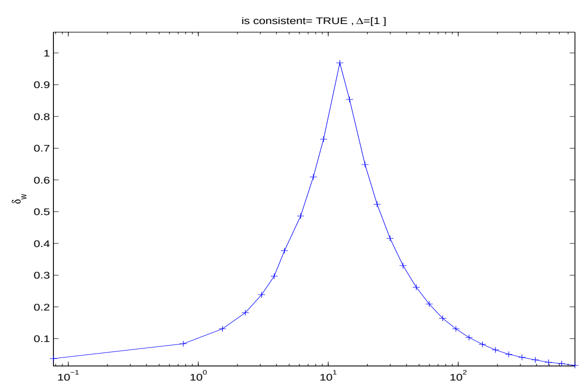

Fig. 8. Uncertainty validation: test case 2

By fitting a transfer function on the resulting magnitude plot one can obtain a candidate for W_w that does not invalidate the model, i.e. it is consistent with the measured data. The result of a validation experiment for the structured uncertainty set Δ_{param} and using a D-scaling method is depicted on Figure 9. In both tests the system was scheduled by Γ.

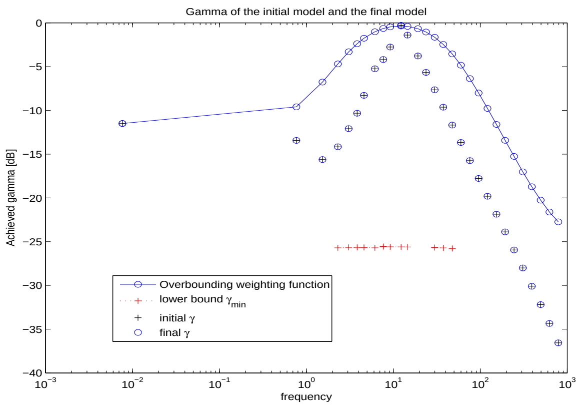

Fig. 9. Uncertainty validation: weight estimation

Simulation results revealed that the bounds obtained by simple relaxation techniques, e.g. nonlinear functional approach, are often too conservative. This is not surprising since the S–procedure is lossless only for some special cases as it was mentioned before. To obtain less conservative bounds with the present techniques, considerable computational power is required. These facts show that while the validation module is able to provide the (in)validation information for a given LPV/LFT model, further research is needed to develop reliable algorithms for an efficient iterative weight selection/controller design algorithm.

D. HL-20 LPV control synthesis and analysis

A very important assessment of the technological readiness level of the developed modeling tool and its practical applicability is the use of the resulting LPV/LFT models for actual control design and analysis. This section summarizes the LPV control and analysis results obtained from using the developed validated LPV/LFT models on the nonlinear simulation model of the NASA HL-20 re-entry vehicle [50], [51].

A very powerful framework for the stability and performance analysis of uncertain and LPV systems is the dynamic integral quadratic constraint (IQCs) approach [32]. Although the IQC framework is mostly suitable for the analysis of uncertain/LPV systems, it can also be employed for LPV controller synthesis purposes if the corresponding IQC multipliers are restricted to be static [35], [42]. These approaches require that the system is represented in the form of a LFT model with the scheduling block depending in a structured way on online measurable parameters. An IQC synthesis and analysis toolbox was also developed within the LPVMAD project and used to synthesize and analyze LPV controllers for the HL-20 [50], [51].

The LPV/LFT models obtained in the previous section are admittedly too complex for LPV synthesis and as a matter of fact, initial trials revealed that performing the controller synthesis directly based on these models was not easy due to the required computation times (about half an hour for determining the performance level for the 2nd order fit LPV/LFT model). This, moreover, lead to numerical problems in the particular controller synthesis procedure implemented in the LPMVAD IQC synthesis toolbox. Additionally, keeping the complexity of the LPV/LFT model low is also crucial for making it possible to analyze the resulting closed-loop system within the IQC analysis environment. Indeed, since the scheduling block that interacts with the IQC synthesized controller is twice the size of the scheduling block that interacts with the plant, the analysis problem will certainly be prohibitive when the size of the plant's scheduling block is large. Therefore, for a quick and easy tuning, a simplified model (named $mod7$) is obtained by simple linear interpolation of the LTI/LFT models at the extreme Mach points of the trajectory. It is remarked that balancing of the scheduling and uncertainty channels is of high importance in order to avoid subsequent numerical troubles during the synthesis.

An IQC-LPV controller was successfully synthesized using the simplified $mod7$ LPV/LFT model. Despite synthesizing a controller on a simplified and possibly inaccurate model, an important step in the design procedure is to analyze the controller on the more complex models of higher fidelity. Thus, the designed IQC-LPV controller was evaluated in an incremental fashion by assessing the resulting frequency and time responses using the for-design model $mod7$, then the 2nd $mod20$ and 3rd $mod30$ order fit LPV/LFT models, and finally the nonlinear time-domain simulations. Further, the use of the LPV/LFT models and the IQC-based analysis tool allows to perform dedicated analysis otherwise not commonly available. For example, Figure 10 shows the robust performance analysis of the designed IQC-LPV controller against delay effects, which can be directly and less conservatively analyzed with the LPVMAD-IQC toolbox. For comparison, a similar analysis using Matlab's robust control toolbox is included –but note that in this case it is necessary to treat the delay as a dynamic LTI perturbation yielding as conservative answers.

V. Conclusions

In this article an integrated LFT/LPV modeling and data-based validation software tool has been presented and its technological readiness demonstrated through its

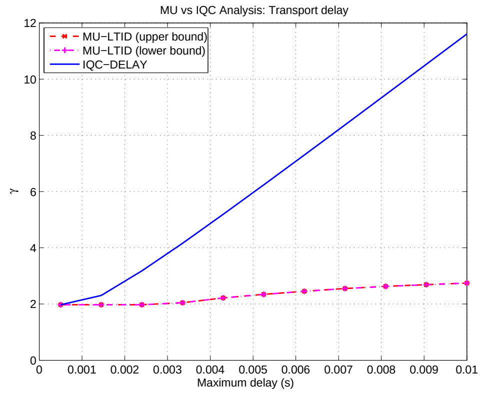

Fig. 10. Robust performance analysis against transport delay.

application for control design and analysis to a nonlinear re-entry system. The development of such a tool is necessary if LPV approaches are to be successfully transferred to industry. The results show that the techniques and tools relying on automated LFT/LPV modeling and combining numerical linearization with symbolic manipulation approaches, are sufficiently mature to be introduced in the control design process within a LPV framework. The conclusion from the assessment also indicates that LPV data-based model validation approaches are not yet at a mature stage for their widespread use within an industrial design process due to their mathematically complex underpinning and a lack of commercial-level tools. Nevertheless, the results from the application of the developed model validation tool to the HL20 model represent a preliminary but successful validation of their potential for further evolution and integration in the modeling cycle of a control development process. The LPV/LFT models obtained with the developed LPV/LFT modeling and validation tool have been successfully used for designing and analyzing LPV controllers for the HL20 using an IQC-based synthesis and analysis toolbox developed in parallel within the LPVMAD study.

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