Application of LPV/LFT Modeling and Data-based Validation to a re-entry vehicle

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In this article an application of LFT/LPV modeling and data-based validation techniques to a re-entry vehicle is shown. This work is part of a European Space Agency project tasked with examining the use of LPV technologies for the control design process of space systems. The application presented serves as an assessment on the technological readiness level of LPV/LFT modeling approaches and data-based validation algorithms, as well as a review of their features, shortcomings and needs. The selected vehicle is the longitudinal nonlinear motion of NASA HL-20 during an approach trajectory from Mach 4.5 down to 1.5.

I. Introduction

THE main modeling representation used in modern control is the linear fractional transformation (LFT) [1, 2], i.e. a feedback interconnection of a nominal linear-time-invariant (LTI) system M and a block-diagonal structured perturbation Δ. This is so because the powerful analysis techniques arising from modern control [3, 4, 5, 6, 7] are generally based on linear-parameter-varying (LPV) concepts and models that rely on representing a system in LFT form. Indeed, LPV systems, and in full generality nonlinear systems given by polynomial or rational expressions, can be represented as LFT models where the parameter-varying, uncertain or nonlinear terms are located in the diagonal Δ operator and what is precisely known and constant in the operator M. Thus, LFT models are LPV models with a special structure. Unfortunately, LPV modeling is not frequently considered as an independent research topic and most of the available literature is only concerned with LPV control/analysis and either assumes the LPV model is given or the model obtained is only of interest for the particular application being studied. In obtaining 'adequate' LPV/LFT models there are two main steps: the first is to obtain the model and the second to validate it.

Generally, control designers use a family of LTI plants at different points of interest throughout the operational and uncertain parameter envelopes in order to obtain the required LPV/LFT model after performing a fitting procedure based on the selected subset of measurable parameters [8, 9]. Part of the reason for the use of this LTI-to-LPV/LFT modeling approach is the considerable time and effort that developing formal LFT models take. Also, the linear approximation used (e.g. the LTI plants) allows the application of linear control and analysis techniques while the explicit use of the uncertain matrix arising from the fitting of the LTI plants imbues the designs and analyses with robustness properties. References [10, 11] proposed an approach where numerical LTI models are obtained first and subsequently they are augmented with explicit symbolic parameter dependency so that in a final step, LFT-based parametric descriptions are automatically generated. This approach has the advantage of connecting the models for design and analysis more directly. In the last 5 years, the importance of using of symbolic processing techniques for LFT manipulation and order reduction has become clear for the LFT modeling community and has sparked the appearance of several approaches and toolboxes [12, 13, 14, 15] that systematize the process of obtaining LFT models.

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As it has been mentioned, modeling is an important precursor to system analysis and controller design. However, before this description can be used by the control engineer, it must be *validated*. As typical in any identification theory, the problem of checking the validity of prior assumptions arises. Model validation [16, 17, 18, 19, 20, 21] provides a systematic way to evaluate the ability of a proposed model to represent observed system behaviors: given experimental data, corrupted by additive noise try to find whether or not this data could have been produced by the combination of the nominal model and some uncertainty in the uncertainty set. Further, the assessment of the quality of a model cannot be decoupled from the purpose for which the model is to be used. The research on identification for control [22, 23, 24, 25, 26, 27, 28, 29] has focused on the design of identification criteria that delivered a control-oriented nominal model. Most of these works addressed deterministic model validation in the robust control domain – typically by representing the model as an LFT between an LTI system and an uncertainty/nonlinearity block— and their results are obtained using operator theoretic approaches based on results from Carathèodory-Fejér interpolation theory [30]. The results shows that in the case of unstructured uncertainty, model validation reduces to a convex optimization problem that can be efficiently solved [31, 32]. Allowing uncertainty with additional structure can make model validation difficult but different approaches exists using weaker conditions [33].

As described in reference [34] due to the specific nature of space systems and a lack of reliable LPV software tools the introduction of LPV techniques in Space has been slow. The European Space Agency (ESA) is addressing this issue by establishing industrial-academia consortiums tasked with developing an industrial LPV control design framework supported with reliable LPV software tools. The LPV modeling, analysis and design (LPVMAD) consortium lead by Deimos Space (Spain) and composed by research teams from the Computer and Automation Research Institute (SZATKI, Hungary), Delft Technical University (The Netherlands) and Leicester University (United Kingdom) was formed to address this need.

The LPVMAD project objectives are: (i) to assess the possibility, needs and impact of LPV techniques in the control design process for space systems; (ii) to propose a control design LPV framework; (iii) to develop reliable LPV tools for modeling, analysis and design in support of such a framework; and (iv) to demonstrate the developed framework and tools in a relevant space system, selected to be an atmospheric re-entry vehicle. Since the re-entry vehicle and mission characteristics are very challenging, the demonstration step is performed in an incremental fashion: from verification in the simplest of the cases (used to provide a transparent analysis on the benefits and limitations of LPV technology) to validation in a more relevant model. The latter validation step is also performed in two incremental stages: the first uses a focused model of the selected benchmark while the second considers the full motion and re-entry flight.

This article presents the technological assessment of the LPV/LFT modeling and data-based validation approaches based on the application of these techniques and tools to the first stage of the LPVMAD validation step. The selected re-entry system is the NASA HL-20 model [35, 36, 37], and for this first validation stage we focus on the longitudinal nonlinear motion during approach trajectory from Mach 4 down to 1.5. The model is sufficiently representative to showcase the challenges found in this type of vehicles as well as the shortcomings and advantages of the LPV/LFT modeling and data-based validation approaches.

The layout of the paper is as follows: Section II provides an introduction to the main LPV/LFT modeling concepts and a review of the LPV/LFT modeling and data-based model validation approaches; Section III details the benchmark used to assess the readiness level of the LPV/LFT modeling and model validation techniques. Section IV and Section V present respectively the application to the benchmark of the LPV/LFT modeling approach and of the LPV/LFT data-based model validation approach. The conclusions are given in Section VI.

II. Review LPV/LFT modeling and data-based validation techniques & tools

The main idea behind using LPV models in lieu of LTI representations is to account for dynamic changes using a scheduling parameter vector, and thus enabling design and analysis of systems subject to time varying trajectories. Also, in comparison to linear time varying (LTV) systems, *LPV approaches do not require anti-causal knowledge on the varying vector* (i.e. LTV systems assume future and past knowledge) since all that they require is that the scheduling parameter is measurable and belongs to a closed set. Indeed, *LPV/LFT systems can be seen as a bridge between linear and nonlinear models* since they are conceptually thought of as a linear model but are embedded with nonlinear characteristics due to their parameter varying nature. Because of this 'linear-but-nonlinear' dichotomy, LPV theory is especially amenable for the definition of a framework that unifies the modeling, analysis and design steps in the control design cycle. Deriving for-design and for-analysis LPV models still require a certain tailoring due to the algorithmic limitations of the LPV synthesis and analysis tools, but the advantage now is that all these models can arise from the same high-fidelity model, which can be obtained from the beginning as an LPV model (and thus the

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connection between the different fidelity models and the analysis results is guaranteed to a larger extent than with current approaches).

As afore mentioned, an LFT is a representation of a system using a feedback interconnection between two operators: $M=[M_{11}\ M_{12};\ M_{21}\ M_{22}]$ and Δ. The operator M represents the nominal, known, part of the system while Δ contains the time-varying, unknown or uncertain components. If the components in the so-called uncertainty operator Δ are scheduling parameters an LPV system is obtained (if they contain states, the model is more appropriately refer as quasi-LPV). A very important property of LFT systems is that their interconnection results in another LFT (e.g. sum, concatenation, etc of LFTs results in LFTs) [12].
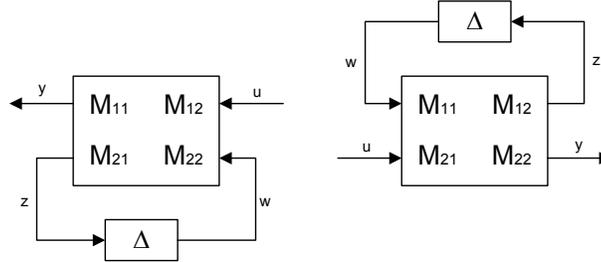

Figure II-1 Lower and upper LFT graphical representation

Depending on the feedback interconnection used, there are two possible types of LFTs, lower (see the left picture in Figure II-1 and Eq. 1) and upper (Eq. 2 and Figure II-1-right):

$$F_L(M,\Delta) = M_{11} + M_{12}(I - \Delta M_{22})^{-1}\Delta M_{21} \qquad \text{Eq. 1}$$

$$F_U(M,\Delta) = M_{22} + M_{21}(I - \Delta M_{11})^{-1}\Delta M_{12} \qquad \text{Eq. 2}$$

The operator Δ is typically norm-bounded for design and analysis (without loss of generality $\|\Delta\|_\infty \leq 1$ by scaling of M), but otherwise unrestricted in form (structured/un-structured) or type (nonlinear/time-varying/constant). It is important to note that *unstructured uncertainty at component level becomes structured uncertainty at system level*. The order of an LFT is the number of parameters, including repetitions, contained in the uncertainty operator Δ (e.g. $\Delta = diag(\rho_1 \times I_2, \rho_2)$ ➔ LFT order of 3). Since many realistic robustness analysis problems can easily result in very high order LFT's, it is vital to have efficient (and automated) tools which can compute minimal, or at least close to minimal, representations of these systems. A review of the main LPV/LFT approaches is given in subsection II-A.

Model validation provides a systematic way to evaluate the ability of a proposed model to represent observed system behaviors: given experimental data corrupted by additive noise try to find whether or not this data could have been produced by the combination of the nominal model and some uncertainty in the uncertainty set. Assuming a LPV/LFT framework for the specific purpose of this work, and without loss of generality, a general uncertainty model can be described by:

$$\begin{bmatrix} \eta \\ y \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \end{bmatrix} \begin{bmatrix} \xi \\ u \\ w \end{bmatrix}; \qquad \xi = \Delta\eta \qquad \text{Eq. 3}$$

Where M_{ij} are LTI systems except for the nominal model M_{23} which is arbitrary. The disturbance w is an element of L_2 and the perturbation Δ can be a structured/unstructured uncertainty operator containing full complex blocks, repeated dynamic scalar and/or repeated real scalar blocks.

Given the uncertain system representation in Eq. 3 and measured input-output pairs *{u(t), y(t)}*, the model validation problem is to find an upper bound γ such that there exist a perturbation $\|\Delta\|_\infty \leq \gamma$ and a disturbance $\|w\|_2 \leq \gamma$ satisfying $e = y - M_{22}u = M_{21}\xi + M_{23}w$. A review of the main model-validation approaches is given in subsection II-B.

A. Review of LPV/LFT modeling techniques and tools

Most authors classify the LPV modeling techniques as linearization-based or quasi-LPV [4, 5] but a broader taxonomy can be used based on the *numerical* or *symbolic* nature of the approach. Simply put, the numerical techniques imply always approximation (at the gain of ease of implementation and development) while the symbolic approaches involve exactness (at the expense of involved manipulations). A brief review of the most common LPV/LFT modeling techniques is given next:

Jacobian: very well known and widespread approach but limited to equilibrium points and linear approximations. The resulting model is a collection of dynamic systems (as opposed to a single dynamic LPV system), which puts

restrictions on the input/output/state structure of the different LTI plants. Inclusion of uncertainty is possible only through additional linearizations (at different parameter values for example) or through ad hoc investigation and fitting of the LTI state-space coefficients in terms of scheduling parameters [8, 9]. The linearization is typically performed numerically but it is also possible to use symbolic manipulations and subsequent numerical substitution [10, 11].

Velocity-based linearizations: it can be used for equilibrium or non-equilibrium points and yields an LPV model, i.e. a single parameter-dependent dynamic system. It still represents a local approximation but with the advantage of not being restricted to equilibrium points. Treatment of parametric and non-parametric uncertainty (structured or non-structured) is possible only through additional ad hoc steps related to polynomial fitting and LFT formulation [4, 39].

State-Transformation: represents an exact transformation in as much that an initial 'symbolic' step is performed to absorb the nonlinearities into the scheduled state-space matrices, but it is an approximated method since a second step is to find certain equilibrium functions (dependent on equilibrium points and with the drawback of not having existence guarantees). The resulting model is a single dynamic entity but as before, incorporation of uncertainty is not direct [3, 6, 7].

Function Substitution: numerical optimization method with the advantage of only requiring a single equilibrium point. Linear programming and least square minimization are used to transform the nonlinearities into terms dependent on the scheduling parameter and affine on the states and/or inputs. The main drawback is a theoretical support not fully developed, for example there is no theoretical study on the effect the choice of the equilibrium point has on the resulting LPV model or subsequent LPV control designs [7, 40, 41].

Parameter-dependent symbolic LFT: symbolic algebra is used initially to obtain a symbolic representation from the nonlinear system. Subsequently there are two paths: first, directly transform it into a symbolic LFT and assign numerical values, or secondly, assign numerical values leaving the scheduling parameter-dependence in symbolic form (and then use first step). Both paths yield an LPV model (not a collection of LTI plants). Incorporation of uncertainty is direct due to the symbolic nature of the approach [7, 12, 13, 14, 15].

Non-Linear LFT: broadly considers all the LPV/quasi-LPV approaches that use symbolic manipulations to yield the LPV model. It is highlighted that once a model is transformed into an LPV system using a symbolic/analytical approach it can be transformed automatically into an LFT. This approach represents an exact LFT transformation as it captures all the characteristics of the nonlinear ordinary differential equations by assuming them as symbolic parameters placed in the Δ operator. The advantage of the approach is that subsequent simplifications are traceable and easy to modify [15].

B.   Review of LPV data-based validation techniques and tools

A review of the most standard data-based validation approaches is given next:

Control-oriented identification. The research on identification for control has focused on the design of identification criteria that delivered a control-oriented nominal model. H_∞ identification, where the modeling error is measured by the H_∞ norm, is among the most investigated set membership methods in the literature in the LTI context. Indeed, H_∞ identification methods deliver uncertainty model sets in a suitable form to be used by well-established robust design techniques based on H_∞ or μ optimization methods. The literature on H_∞ identification is now very extensive, considering time and frequency domain measurements and under different prior assumptions [22, 23, 24, 25, 26, 27, 28, 29].

Model-validation methods. Before a model description can be used by the control engineer, it must be *validated* based on experimental data. The only thing that can be actually done is to check if prior assumptions are invalidated by the data, i.e. evaluate if no system exists consistent with data and assumptions. To validate a model set through experimental data, the consistency condition plays an essential role in both stochastic [16] and deterministic frameworks.

Indeed, motivated by the need of constructing system models for robust control design, validation of uncertain models that are of a worst case, deterministic nature have received considerably attention. For example, Sokolov [42] reported a control-oriented model validation and error quantification method in an L_1 setup while Smith and Doyle [18] initiated a frequency domain approach to uncertain models of a linear fractional structure. Their formulation was fairly general, but the resulting validation test was difficult to implement. Later, Poolla et al. [17] considered models with an additive uncertainty and adopted a time domain approach. Model validation using time-domain data was also addressed by Chen and Wang [43]. They showed that under certain assumptions the problem can be reduced to a class of convex programs, i.e. LMI problems, to which readily implementable algorithms are available. Research work has been further pursued by Smith and Dullerud, Rangan and Poolla, on validation problems pertaining to sampled-data systems [44, 40, 19]. The problems considered in these references also admit uncertain models of an additive structure

and are solved in time domain as LMI problems. All the models possess a structure linear in the modeling uncertainty, that appears to be a key condition for solutions via convex optimization. Model validation using mixed frequency-domain and time-domain data was investigated by Xu et al. [31]. Quite recently, nonlinear model validation was addressed by Dullerud and Smith, [33], and LPV model validation by Sznaier and Mazzaro [32].

LFT model validation. Most of the works above addressed deterministic model validation in the robust control domain –typically by representing the model as an LFT between an LTI system and an uncertainty/nonlinearity block—, and their results are obtained using operator theoretic approaches based on results in Carathèodory-Fejér interpolation theory [30]. The results shows that in the case of unstructured uncertainty entering the plant as an LFT, model validation reduces to a convex optimization problem that can be efficiently solved [31, 32]. Allowing uncertainty with additional structure can make model validation difficult. In the case of structured uncertainty and LFT dependence the problem leads, in general, to bilinear matrix inequalities (BMI). However, (weaker) necessary conditions for consistency in the form of LMIs are also available.

Motivated by analysis results using nonlinear functionals and the S-procedure [45] and results on integral quadratic constraints (IQCs) [46], several well known and also new (e.g. sector bounded nonlinear) perturbation formulations have been recast in a novel model-validation framework [33]. This novel formulation presents an extension of the model validation in terms of perturbations described as IQCs or more general nonlinear functionals. The basic idea is that by using functionals to define the allowed behavior of the unknowns in the model validation problem, we can express precisely when a model will be invalidated in terms of the simultaneous solution to a list of functional inequalities

It is a common characteristic of these methods that they are based on discrete-time models or an implicit discretization of the model as in [44]. For the qLPV case, due to the LFT structure, it is possible to set a model where the generalized plant is time-invariant therefore, in principle, this fact does not introduce further difficulties in the solution of the problem. However, it is not completely clear yet how the discretization affects the result of the validation algorithm.

ν-gap model validation. The Vinnicombe (or ν-gap) metric notion [47] has attracted much attention particularly in robust control and system identification. The ν-gap metric measures the aperture of two closed Hilbert sub-spaces representing the bounded input-output pairs of two linear (possibly unbounded) operators. Several attempts have been made to extend the idea of the ν-gap metric to nonlinear systems. An indirect closed-loop nonlinearity measure using the ν-gap metric and the quasi-LPV representation was proposed in [48].

The main interest of the ν-gap metric is its use in a range of robust stability results. One of these results relates the size of the set of robustly stabilizing controllers of a ν-gap uncertainty set to the size of the uncertainty set [49]. Gevers et al. presented a method for model validation for robust control and controller validation in a prediction error identification framework by introducing a measure for the size of a prediction error uncertainty set that is directly connected to the size of a set of controllers that stabilize all systems in the model uncertainty set [50]. That makes possible to compare uncertainty sets for robust control design, leading to control-oriented validation objectives. A closed-loop time-domain model validation in the ν-gap metric was reported in [20], where LMI sufficient conditions were given for model consistency along with LMI necessary and sufficient conditions for invalidation with respect to a special class of input/output noise.

III.   LPVMAD benchmark: longitudinal motion of the NASA HL-20 re-entry vehicle

In terms of the selected system relevancy for LPV modeling, the challenging aspects are: (i) the nonlinear character of the model and the inclusion of practical implementation issues (such as time delay, magnitude and rate saturation), (ii) the aerodynamic database and nonlinear equations of motion that allows for relevant (i.e. physical, system-based motivated) introduction of uncertainty, (iii) the wide range of dynamic variation, which poses a challenging problem in capturing them in a single model.

The NASA HL-20 lifting-body vehicle was proposed as a substitute of the U.S. Space Shuttle Orbiter. Although finally de-commissioned, many research efforts in the experimental and computational fields were performed to develop a fairly detailed baseline aerodynamic database for the complete speed range of the vehicle [35, 36, 37]. The two main questions for which the baseline vehicle and aerodynamic database were developed addressed the controllability of the vehicle in the high-heat/high-hypersonic re-entry phase from orbit, and its controllability in the lower atmospheric phase –with special emphasis to flying qualities during this lower phase. The latter low atmospheric phase is especially relevant for the purposes of the LPVMAD project since:

- Return flights are typically unpowered (thus, almost no margin of error for correction).
- Re-entry vehicles typically have low lift-to-drag ratio at the low speeds used in approach phase (i.e. thus aerodynamic stability and maneuverability is poor when most required).

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- The large flight envelope of a re-entry vehicle constrains the design trade-offs required by the GNC system, which must be valid for the wide dynamic variations (e.g. from Mach around 20 down to low subsonic speeds). Thus, the stability and aerodynamic characteristics of the vehicle are again poor at low speed since it needs to satisfy more stringent high-speed constrains.

For the purposes of this work, an implementation[§§] of the nonlinear equations of motion for the longitudinal motion of the HL20 vehicle is used together relevant aerodynamic information (all taken from NASA's reports [35, 36, 37,]). The aerodynamic database covers a wide range of performance variation: M∈[4.0→1.5], he∈ [30→12] Km, α∈ [-2, 30] degrees and β∈[0, 2] degrees. Similarly, an auto-land trajectory [35] is used starting at Mach 4 and 31 Km in altitude and ending near Mach 1.5 and approximately at an altitude of 10 Km, see Figure III-1. The profile of the given reference trajectory follows typical autoland profiles for landing-able re-entry vehicles (e.g. HL-20 and Space Shuttle). It has been defined based on information from the previous NASA references but modified to provide desirable performance & robustness objectives.
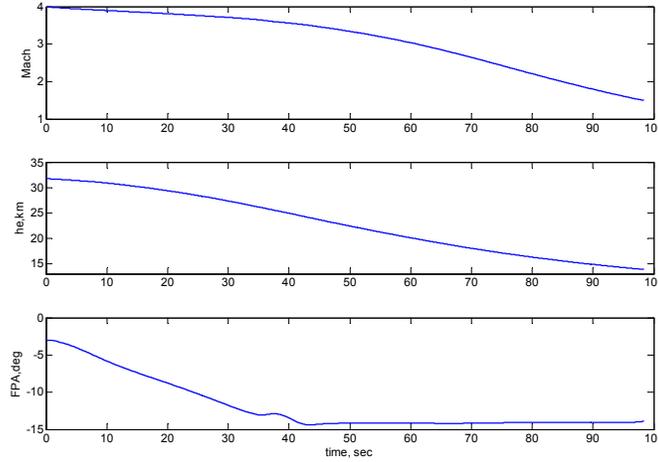

Figure III-1 HL-20 reference approach trajectory

A. NASA HL-20 longitudinal equations of motion and aerodynamic database

The equations of motion are formulated under the following assumptions:
- Aero-elasticity of the vehicle is neglected, thus the vehicle is considered as a rigid body.
- Translational and rotational motions are considered coupled (due to the increase dynamic pressure as the vehicle flies down, which couples rotation and translation).
- Lateral/directional motion coupling is neglected (a fairly standard assumption in atmospheric flight).

For convenience, the dynamic equations of the translational motion are derived using spherical position and velocity coordinates. R denotes the distance from the centre of gravity of the vehicle to the centre of the Earth, V represents the ground speed, γ is the flight-path angle, α is the angle of attack, σ is the bank angle, m is the aircraft mass, and the pitch rate q is given with respect to the body-reference-axis (BRF). Earth is assumed to be an ellipsoid with mass symmetry about the polar axis and gravitational constant μ_E ($g = \mu_E / R^2 \approx 9.81\ m/s^2$):

$$\dot{V} = \frac{F_{wx}}{m} - \frac{\mu_E}{R^2}\sin\gamma \qquad\qquad \text{Eq. 4}$$

$$\dot{\gamma} = \frac{-F_{wz}}{Vm} - \frac{\mu_E}{R^2 V}\cos\gamma + \frac{V}{R}\cos\gamma \qquad\qquad \text{Eq. 5}$$

$$\dot{R} = V\sin\gamma \qquad\qquad \text{Eq. 6}$$

$$\dot{\alpha} = q + \left(\frac{F_{wz}}{gm} + \cos\gamma\right)\frac{\mu_E}{R^2 V} \qquad\qquad \text{Eq. 7}$$

$$\dot{q} = \frac{M_y}{I_{yy}} \qquad\qquad \text{Eq. 8}$$

[§§] The implemented HL-20 system is a quite more advanced, and representative, than the Mathworks publicly available model [38] which uses the aerodynamic polynomial simplifications given in [36] and is valid only for the Mach range ∈ [0.08, 0.6].

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For realistic performance simulation purposes, the aerodynamic coefficients are directly obtained from the aerodynamic database from $M \in [0, 4.5]$. The stability derivatives are formed by around 68 nonlinear look-up tables (LUTs) dependent on Mach number and control surface deflection, thus a simplified version is used considering only the longitudinal dynamics, no ground effects and no landing gear deflection. These LUTs are combined as function of the angle of attack (always a third order polynomial) to obtain a specific stability derivative. For example, the static-contribution of the lift coefficient is given by:

$$Cl0 = Cl0A0(M) + \alpha \cdot (Cl0A1(M) + \alpha \cdot (Cl0A2(M) + \alpha \cdot Cl0A3(M))) \qquad \text{Eq. 9}$$

And for CLBFUL (lift coefficient derivative due to the upper left body flap δ_{BFUL}):

$$ClBFUL = ClBFUL0(M, \delta_{BFUL}) + \alpha \cdot (ClBFUL1(M, \delta_{BFUL}) + \alpha \cdot (ClBFUL2(M, \delta_{BFUL}) + \alpha \cdot ClBFUL3(M, \delta_{BFUL}))) \qquad \text{Eq. 10}$$

Subsequently, the stability derivatives are added to obtain the longitudinal aerodynamic coefficients using standard implementation rules [35].

B. NASA HL-20 actuation and sensor systems

The HL-20 lifting body aero surface configuration is shown in Figure III-2. The available control surfaces are upper left and right flaps (DUL and DUR), lower left and right flaps (DLL and DLR), wing left and right flaps (DEL and DER) and rudder (DR). The deflection is positive downward for the upper/lower flaps (+TED and -TEU), away from the vehicle for the wing flaps (+TED) and to the left for the rudder (+TEL).
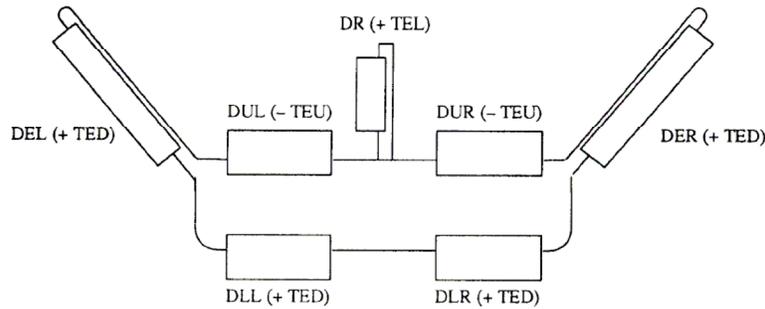

Figure III-2 HL-20 control surface configuration, view from rear (from NASA reference [35])

A control surface mix logic is implemented in the nonlinear model depending only the longitudinal 'pilot/autopilot commands' (elevator deflection δ_{ele} and speed-brake deflection δ_{sbk}) and Mach number:

$$DEL = DER = \delta_{ele}$$
$$DUL = DUR = f_1(\delta_{ele}, \delta_{sbk}, M)$$
$$DLL = DLR = f_2(\delta_{ele}, \delta_{sbk}, M) \qquad \text{Eq. 11}$$
$$DR = 0$$

The output from the control-mixer is passed to the actuation system consisting, for each channel, in a series connection of 2nd order dynamics, magnitude limiter, rate limiter and time delay (of 0.005 seconds, implemented as a 1st order Padé approximation), see Figure III-3.
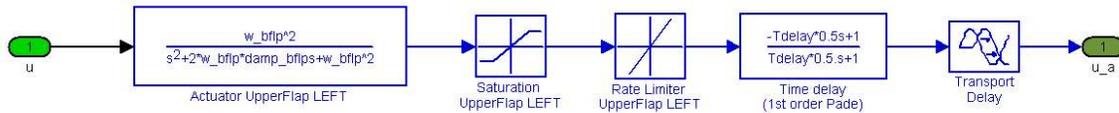

Figure III-3 HL-20 implemented actuation system

The sensor system is implemented as first order colored filter with a sensor-specific corner frequency K, bias b and Gaussian white noise ν (a Matlab band-limited white noise block with unity noise power and a sample time of 0.01 seconds is used to generate ν).

C. Uncertainty

In order to allow a relevant examination of the LFT/LPV modeling techniques developed within the LPVMAD project, as well as those for design and analysis [51, 52, 53, 54], parametric and aerodynamic uncertainty models (additive, multiplicative and inverse multiplicative) are introduced in the nonlinear implementation of the model. The

full uncertainty matrix $w_\Delta = \Delta\, z_\Delta$ is of dimension 16 with 8 different uncertainties: 5 parametric (corresponding to center of gravity coordinates, moment of inertia, mass [x_{CG}, y_{CG}, z_{CG}, I_{yy}, m]) and 3 aerodynamic (corresponding to the longitudinal coefficients [C_L, C_D, C_M]).[***]

D. Analysis Open Loop LTI Plants

Linear-time-invariant (LTI) models are obtained using a tailored numerical linearization tool, which allows obtaining LTI models of the HL-20 along a selected trajectory given a Mach number and an uncertainty set. Table 1 shows the eigenvalues for the nominal and ±50% uncertainty LTI plants at two trim points: Mach 4.0 and 2.0.

Table 1 HL-20 modal analysis: LTI plants at M=4 and M=2

		Δ=-50%	Δ=0	Δ=+50%
Mach 4		-0.0505 + 1.3599i	-0.0267 + 1.3024i	-0.0068 + 1.2404i
		-0.0505 - 1.3599i	-0.0267 - 1.3024i	-0.0068 - 1.2404i
		-0.1846	-0.0123 + 0.0399i	-0.0124 + 0.1889i
		0.1636	-0.0123 - 0.0399i	-0.0124 - 0.1889i
		-0.0005	0.0015	-0.0003
Mach 2		-0.3578 + 3.9875i	-0.1632 + 4.0249i	0.0323 + 3.9653i
		-0.3578 - 3.9875i	-0.1632 - 4.0249i	0.0323 - 3.9653i
		-0.1757	-0.0097 + 0.0423i	-0.0163 + 0.1839i
		0.1515	-0.0097 - 0.0423i	-0.0163 - 0.1839i
		-0.0030	-0.0086	-0.0025

Notice that the nominal system is unstable at the higher Mach and becomes stable as it flies down the trajectory. Similarly, the addition of uncertainty has a dramatic effect changing the system stability properties for both Mach numbers: unstable in the low Mach for both uncertainty sets while for the higher Mach, the positive Δ makes the system stable and the negative Δ increases the degree of instability (measured as distance to the imaginary axis). This eigenvalue analysis showcases the challenge and representativeness of the selected benchmark.

The longitudinal dynamics consists mainly of two modes: (i) short period mode and (ii) a lightly damped, low frequency phugoid mode, which are indicated *Figure III-4*. For example, at Mach 0.5, the natural frequencies of these modes are $\omega_{phugoid}$= 0.047 radians/sec and $\omega_{short-period}$ = 4.66 radians/sec.
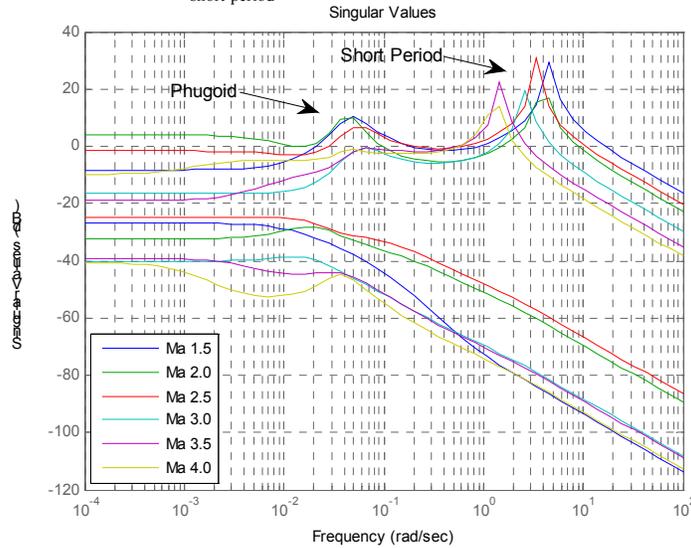

Figure III-4 HL-20 singular values open loop plant (with no actuator and sensor)

[***] The results presented subsequently, and those in the associated references [51, 52, 53, 54], pertain the current implementation and *serve to validate the methods* but it is highlighted that the specific numerical results are system-dependent. For example, it is known by the authors that application of the LFT modeling approach presented in Section IV to a different vehicle, and even to a more developed implementation of the HL20, yielded a significant more conservative LPV model. Further research in this aspect is being carried out.

IV. LPV/LFT modeling of the benchmark system

In this section the developed LPV/LFT modeling approach and its application to the previous system is given.

A. LPV/LFT modeling techniques and tools

The objective of the LPV/LFT modeling approach is to capture in a single LPV model, with an LFT structure, the dynamic behaviour of the HL-20 along the defined trajectory as Mach changes but also considering the changes due to the parametric uncertainty (mass, moment of inertia, center of gravity and aerodynamic coefficient knowledge). This objective is represented in *Figure IV-1*.
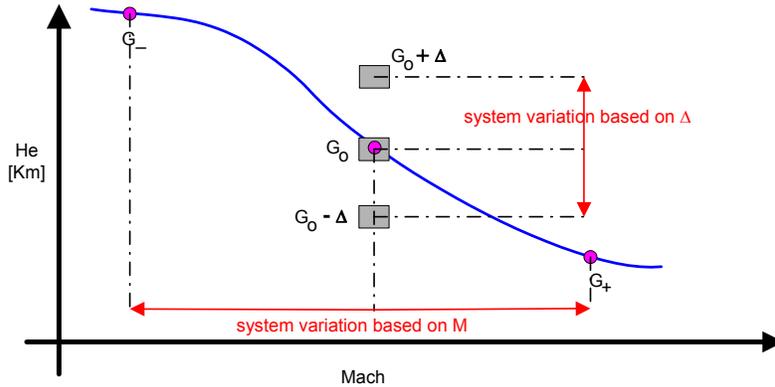
Figure IV-1 LFT numerical state-space system Mach and Δ coverage

The approach and associated software tool are developed within the LPVMAD framework [34] and allows a GNC engineer to automatically obtain a numerical LTI state-space plant, in an LFT format, given a Mach number and a value for the uncertainty set. From the user perspective it is a three-step approach: (i) numerical trimming of the vehicle at the desired Mach and Δ values, (ii) numerical linearization using the trim values from the previous step (yielding an uncertain LFT model) and (iii) symbolic fitting of selected scheduling rule across dynamic variations (yielding an LPV model, in the scheduling parameter, in LFT form).

The first step is necessary for accessing any of the two subsequent steps as it provides the required (pseudo)-equilibrium information to perform the numerical linearizations of the second step, required in turn for the third step. It uses a general Simulink© template for re-entry vehicles, which for the present purposes is particularized to the HL-20 benchmark (i.e. contains the corresponding actuator, sensor, environment and aerodynamic database models). This implementation is characterized for using an LFT form that captures the introduction of the uncertainty in a physically meaningful manner. The second step uses the standard Matlab© numerical linearization tools and, thanks to the aforementioned LFT-based Simulink© implementation, the resulting numerical LTI state-space plants are directly obtained in an LFT format that contains the physical input/output channels for the uncertain parameters. This avoids the need to fictitiously and conservatively introduce, over bound, or otherwise capture the parametric uncertainty effects. Further, as it will be shown later, this step allows getting a single numerical LFT model –albeit conservative of course depending on the nonlinear characteristics of the system— that captures the parametric uncertainty around the selected Mach in a single shoot (without requiring multiple trimming and linearizations around that Mach for different combinations of parametric uncertainty values). The third step performs a symbolic least square fitting, of the LTI-LFT plants obtained before, across Mach-variations to finally obtain a full-envelope numerical LPV-LFT model with a symbolic uncertainty matrix Δ that captures both the parametric and dynamic variations.

The approach developed is quite general and can be readily used for other type of re-entry vehicles since the Simulink implementation captures the possible parametric uncertainty in the moment-transfer and the equations of motion, which are well-known and common to all vehicles: mass, moment of inertia, center of gravity position. The aerodynamic database is not transformed into a Simulink-based LFT due to the wide variations in vehicles, thus the need for the third step above. The Simulink general template used in step (i) and (ii) only captures aerodynamic uncertainty by introducing multiplicative uncertainty models in the aerodynamic coefficients. Obviously, the uncertain parameters are always normalized between ± 1 as usual in LFT modeling.

The models obtained from using the first two steps have been successfully used in the assessment of LPV loop-shaping controller synthesis [51] and optimization worst-case approaches [54], while the final (single) LPV model arising from using the three steps has been also successfully used to assess IQC-based synthesis and analysis

techniques [52, 53]. In subsection IV-B, the results from using the proposed LPV/LFT modeling approach and tool are described.

It is noted that the approach differs to previous LPV/LFT modeling approaches [10, 11, 12, 13, 14, 15] in that it combines symbolic LFT manipulations and standard (numerical) Matlab/Simulink implementation to automatically get LPV models in LFT form. The method has parallels, in as much as that both attempt to use automated Simulink-based LFT modeling tools, with that proposed in [55] and indeed combination of these two methods can possibly result in a powerful Simulink-based LFT modeling tool readily usable by GNC engineers. A snapshot of the main Simulink (refer to as MDL from now on) template file for re-entry vehicles is given in *Figure IV-2*. As mentioned before, the importance of this MDL template file is that it generalizes the implementation of nonlinear re-entry systems while providing a high degree of flexibility to the user. Despite the advantages it is highlighted that the well-known issue of the trim effects (endemic anyhow to all other LFT approaches based in linearization techniques) is still present and deserves further research.
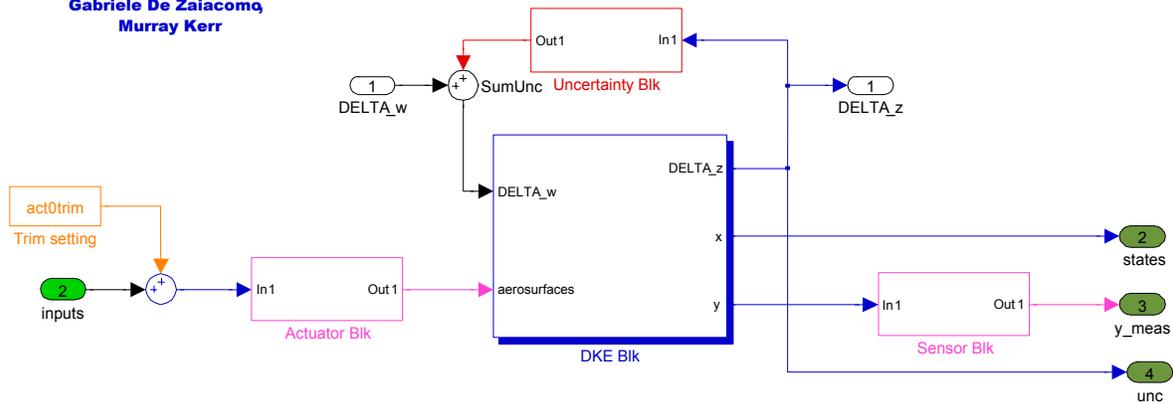

Figure IV-2 LPVMAD general open-loop nonlinear Simulink shell for re-entry vehicles

B. Application to the HL-20 benchmark
In this subsection the application of the above approach is showcased on the selected benchmark.
1. Obtaining numerical LTI plants with parametric uncertainty in LFT form

The LFT-LTI models directly obtained from the numerical trimming and linearization algorithms have a dimension 29x23 (including states, outputs, inputs and 16x16 parametric uncertainty input/output channels). The states are given by x=[q, α, V, γ, R], the inputs by u=[w_Δ δ_{elev}] and the outputs by y= [z_Δ, q, nx, nz, α, γ, $qdyn$, M, he] where *nx, nz* are respectively the longitudinal and normal accelerations and *qdyn* is the dynamic pressure. The parametric uncertainty matrix $w_\Delta = \Delta z_\Delta$ is of dimension 16 with 8 different uncertainties: 3 longitudinal aerodynamic coefficients [C_L, C_D, C_M] and 5 parametric [x_{CG}, y_{CG}, z_{CG}, I_{yy}, m].

Figure IV-3 shows the trim elevator deflection results for different Mach numbers and nominal uncertainty (i.e. $\Delta=0_{16\times16}$) along the selected trajectory. It is noted that overall the changes are smooth and indicate the need for higher, in absolute sense, deflection of the elevator in order to maintain the trajectory (recall, that the vehicle is unpowered).
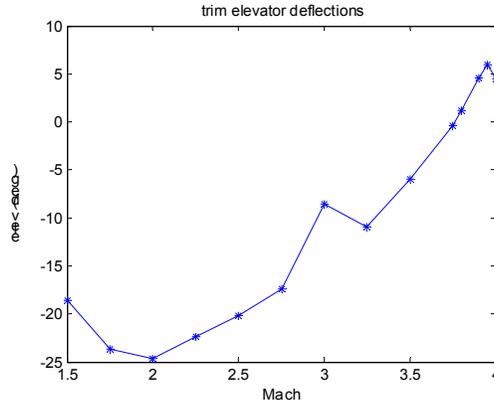
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Figure IV-3 Elevator deflection trim values

The obtained trim values are used together the Simulink file from *Figure IV-2* to numerically obtain the LTI plants at given Mach number and uncertainty values. In order to ascertain that the LTI plants in LFT format have captured adequately the physical uncertainty input/output relations, an eigenvalue assessment is performed.

Figure V-4 shows a comparative of the eigenvalues for two Mach numbers, at 3.8 (top row) and 1.75 (bottom row), and two levels of uncertainty, -40% (left column) and +40% (right column). The poles marked with 'x' are those for the LFT-LTI plant obtained numerically setting the uncertainty to zero, we will call it the nominal system LTI_{NOM}. Closing manually the LFT formulation, by application of Eq. 1 or Eq. 2, of LTI_{NOM} around a selected uncertainty value yields the perturbed system $LTI_{NOM}(\Delta)$ –its poles indicated by 'o'. The latter eigenvalues should be similar, if not equal, to those from the LFT-LTI system obtained numerically from the LPVMAD modelling tool when the uncertainty is set to the same Δ as for the manual LFT, marked with '□'. It is clear from *Figure V-4* that this is the case, in spite of the large changes in the poles across Mach and Δ variations[†††].
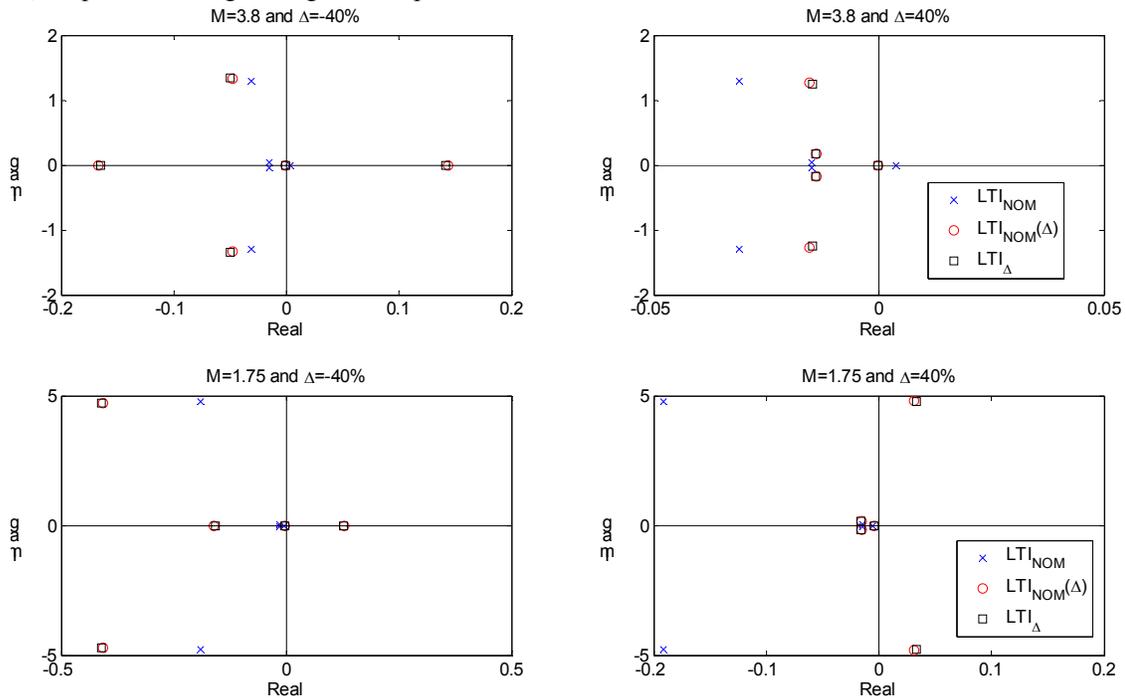

Figure IV-4 Poles comparison LTI$_{NOM}$, LTI$_{NOM}$(Δ) & LTI$_\Delta$

[†††] Again it is remarked that these results allow validating the presetned LFT modeling method but that results for other systems or different Simulink implementations might yield more conservative LFT models whose usefulness will depend on the tasks for which the models are used (design allows for a more crude, conservative LFT model while analysis demands a more higher fidelity model).

2. From family of numerical LFT to a single LPV model in LFT form

The next step is to somehow combine the obtained set of LFT-LTI plants into a single LFT-LPV model. The most standard approach is to fit the state-space systems with a symbolic polynomial based on the selected scheduling variable and then apply again symbolic manipulation tools [12, 13, 14] to get the LPV model in LFT format (where the scheduling variable is now included in the Δ operator).

As mentioned before, the LFT-LTI models dimension is 29x23 (5 states, 7 measured outputs, 1 inputs and 16x16 parametric uncertainty input/output channels). This means that fitting even a linear combination of two of the LFT-LTI systems based on Mach will likely yield a very large Mach-dependent dimension of 23 (unless for example, the uncertainty channels are removed and the output dimension is reduced to only three signals due to the specifics of the control design objectives [51]).

In the HL-20 case, the approach used is to fit, coefficient by coefficient, a family of 4 plants obtained at Mach = [4.0, 2.75, 1.75, 1.5] with a symbolic 2nd and 3rd order polynomials in Mach using a symbolic least square procedure. This yields a model with *20 and 30 repetitions respectively in Mach* (not counting the 16 from the parametric uncertainty set). The resulting system is a single LPV model in LFT form.

The validity of this LFT-LPV model is again assessed by comparing the eigenvalues and frequency responses of the LFT-LTI systems extracted from the LFT-LPV model (by fixing Mach and parametric uncertainties at specified values) with those corresponding to the LTI systems obtained directly from the nonlinear system (i.e. by trimming and linearizing at the same Mach and uncertain values using the Simulink implementation). Furthermore, as it will be seen in the next subsection, the LFT-LPV model was also validated using the LPVMAD uncertainty validation tool.

Figure IV-5 shows again an eigenvalue comparative at specific Mach (same as before) and Δ values (±20%). The LTI model numerically obtained from the MLD file is marked by 'x', the LFT-LPV models using the 2nd and 3rd order fitting approximations are marked respectively by 'o' and '□'. It is seen in the figure that the difference between the two LFT-LPV systems with respect to the LTI is difficult to assess (in terms of which one is better) but that both show quite approximately the same values as the numerical LTI –which should be considered the baseline.
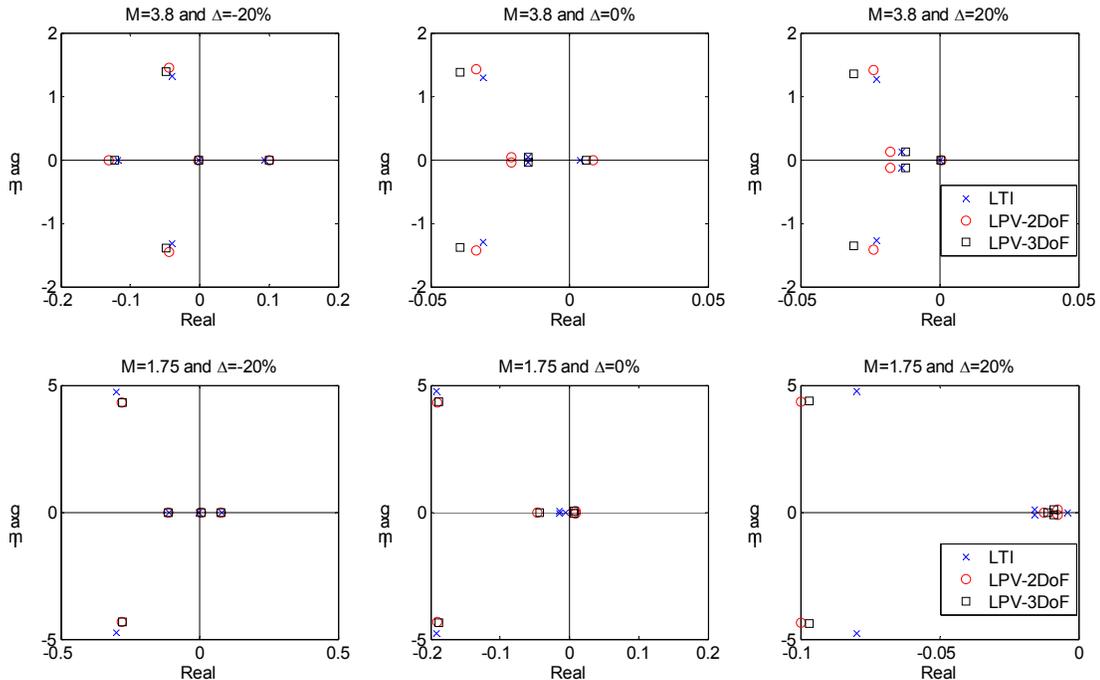

Figure IV-5 LTI, LPV-2DoF fit & LPV-3DoF: eigenvalue comparison

In order to better determine the effect of the approximation, Figure IV-6 shows the variations with respect to Mach of the natural frequency *Wn* (left column), damping coefficient (middle column) and pole value (right column) for each of the five states. It is now possible to observe that the LFT-LPV 3rd order model is, as expected, closer to the values of the LTI model –although the differences are minimal with respect to that from the 2nd order LFT-LPV.
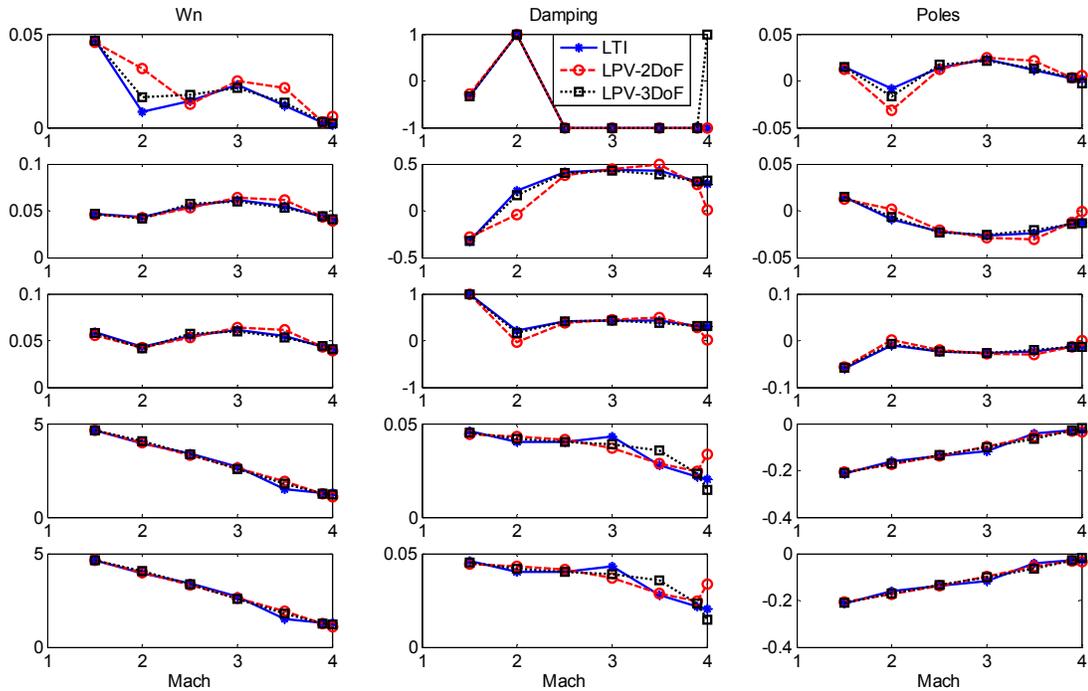
Figure IV-6 LTI, LPV-2DoF fit & LPV-3DoF: natural freq (Wn), damping & pole comparison

A similar coefficient-by-coefficient assessment of the *A, B, C, D* matrices for the same systems as above was also performed and yielded similar results. Finally, a comparison is performed based on the frequency domain response for the same three systems, shown in Figure IV-7. This yielded a similar conclusion: the LFT-LPV models are both adequately capturing the LTI dynamic characteristics. Since the 2nd order fit model is smaller in Mach-dimension this LFT-LPV model will be used for the assessment of the LPV data-based validation (next section) and IQC-based design and analysis approaches [52, 53][‡‡‡].

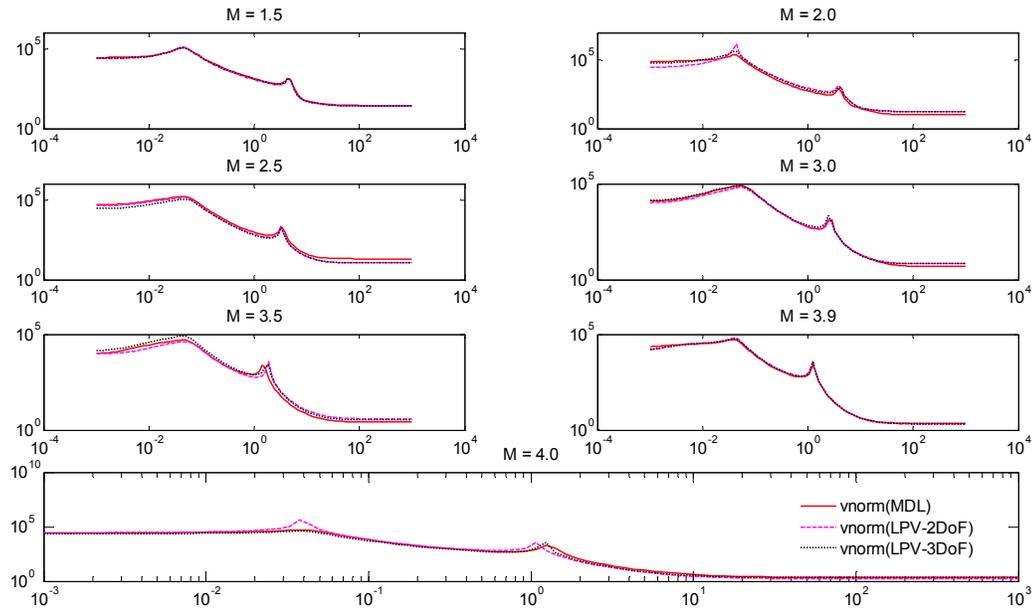

Figure IV-7 LTI, LPV-2DoF fit & LPV-3DoF: frequency response (norm)

[‡‡‡] For the LPV IQC design [52], these 2nd and 3rd other models yielded conservative LPV controller designs. Further experimentation with higher-fitting models (4th and 5th) enabled much improved controllers. Since the differences above are not noticeable, in terms of eigenvalue and frequency responses, this issue clearly deserves further research.

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V. LPV/LFT data-based model validation of the benchmark system

Within the LPVMAD project a LPV/LFT data-based model validation toolbox has been developed. The primary function of the toolbox is to provide a computationally tractable method to (in)validate an LFT/LPV model, i.e. to test that the acquired measurement data does not contradict the given uncertainty structure with given weights W_Δ, W_w associated to a nominal LPV plant. A secondary function of the developed toolbox is to facilitate a joint uncertainty modeling and controller synthesis process by simultaneously tune W_Δ and W_w in the set of all admissible weighting functions that satisfy the model validation problem.

It is highlighted that model validation techniques are based on highly complex mathematical tools and thus, the results presented here represent a successful but preliminary examination of the potential for the introduction of these techniques into the LPVMAD control design framework. Subsection V-A presents the theoretical underpinning of the used model validation approach while subsection V-B shows the results of their application to the HL20 problem.

A. LPV data-based model validation and weight-design problem

The LPV data-based model validation procedure uses frequency-domain data for systems represented by Eq. 3 with Δ containing LTV/LTI uncertainties (structured or unstructured) and with noise $w \in L_2$. Thus, the technique relies on the assumption that a basic LFT model structure has been already determined: a nominal LPV plant model M with given scheduling variables, a disturbance w and uncertainty model Δ. It then tries to answer if the LFT model passes the invalidation test.

Motivated by analysis results using nonlinear functionals and the S-procedure in [45], and results on integral quadratic constraints (IQCs) in [46] several well known perturbation formulations are recast in a novel framework in [33] where an extension of the model validation is given by describing the perturbations as IQCs or more general nonlinear functionals. The basic idea is that by using functionals to define the allowed behavior of the unknowns in the model validation problem, it is possible to express precisely when a model will be invalidated in terms of the simultaneous solution to a list of functional inequalities.

In general it is difficult to compute when the set of solutions is empty, however, using the S-procedure a sufficient condition for model invalidation can be expressed in terms of a single functional inequality with multipliers. Then in the case of quadratic functionals, the latter inequality can be exactly expressed as a semidefinite programming problem. Furthermore, in the more general setting of multinomial functionals a sufficient condition can also be computed in terms of semidefinite programming, providing computable conditions for invalidating a wide range of robust control models from data.
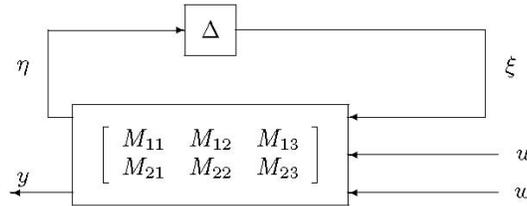

Figure V-1 **LFT model-validation setting**

The LFT model-validation framework considered is shown in *Figure V-1*, where systems M and Δ are discrete-time and causal, obeying to the equations :

$$\begin{aligned} \eta &= M_{11}\xi + M_{12}u + M_{13}w \\ y &= M_{21}\xi + M_{22}u + M_{23}w \\ \xi &= \Delta\eta. \end{aligned}$$

Eq. 12

The input w is considered to be an unknown exogenous signal, taken from a known bounded set while the other input u is known or measured and might correspond to the excitation input in an identification experiment. In the model validation problem, the output signal y is also considered to be known or measured. Measurement noise is modeled as a component of w, and in any experimentally based problem M_{22} can be assumed to be left invertible. All the given signals are finite sequences corresponding to measurements on a finite horizon of length T.

The unknowns in the setup are the perturbation Δ and the input disturbance w, which are only assumed to be constrained to particular sets. Constraining the function Δ is equivalent to constraining the ordered pair (ξ, η), thus one can put constraints on w, η and ξ to satisfy $w \in \Gamma_w$ and $(\xi, \eta) \in \Gamma_\Delta$, rather than working with Δ directly. Given a quadratic functional $G_{\bullet,i}(\zeta) = \zeta^* G_{\bullet,i} \zeta + \beta_{\bullet,i}^* \zeta + \gamma_{\bullet,i}$ (a special case of the constraint is when the vector β and the scalar γ are zero, in which case it is known as an IQC) the signal sets Γ_w and Γ_Δ are defined by:

$$\Gamma_\bullet = \{\zeta \mid G_{\bullet,i}(\zeta) \geq 0, \quad i = 0,\ldots,d_\bullet\} \qquad \text{Eq. 13}$$

Since model validation methods can only invalidate models it is enough to search for sufficient conditions for invalidation as they are the only conditions which lead to definitive statements about model quality.

The key point exploited by the method is that the composition of an affine functional and a quadratic one is quadratic. Thus, if G is a quadratic functional in the variable π, then $H(\theta)=G(\pi(\theta))$ is a quadratic functional in θ.

Theorem: Given the data *(u, y)* and the quadratic functionals H_j if there exist scalars $\tau_i \geq 0$ such that the inequality

$$H_0(\theta) + \tau_1 H_1(\theta) + \cdots + \tau_N H_N(\theta) \leq 0 \qquad \text{Eq. 14}$$

holds for all θ, then the model is invalidated.

This result is exact when there are only two functionals, and a value of the parameter θ can be found that makes H_1 positive. Moreover the test of the Theorem is readily converted to a LMI feasibility problem.

As mentioned before, the LPVMAD toolbox also facilitates the determination of weighting functions W_Δ, W_w and tests if the model with these computed weights can be invalidated. There is a wide range of freedom in balancing between the elements of the uncertainty sources (by tuning of W_Δ and W_w), and the final choice influences the achievable closed-loop performance. The goal of the weighting function uncertainty modeling is to improve robust performance while maintaining the validity of the model, see *Figure V-2*.
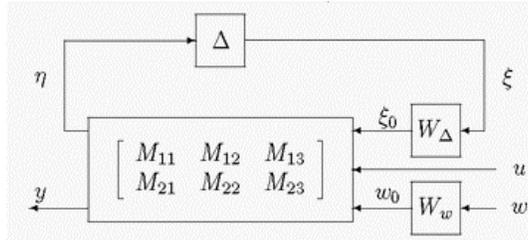

Figure V-2 Setup for LFT data-based validation

Model validation weight-design problem:

Given measurement data *(u, y)*, find weighting functions W_Δ and W_w

$$\begin{aligned} W_\Delta &= diag\{W_{\Delta_1} I_{\xi_1},\ldots,W_{\Delta_\tau} I_{\xi_\tau}\}, \\ W_w &= diag\{W_{w_1},\ldots,W_{w_{n_w}}\} \end{aligned} \qquad \text{Eq. 15}$$

such that there exist a perturbation $\|\Delta\|_\infty \leq 1$ and a disturbance $w \in L_2$ such that

$$e = y - M_{22}u - \begin{bmatrix} M_{21} & M_{23} \end{bmatrix} \begin{bmatrix} \xi_0 \\ w_0 \end{bmatrix}. \qquad \text{Eq. 16}$$

B. Application results

For the assessment of the LPVMAD data-based model validation techniques and tools, the longitudinal nonlinear HL-20 system described in Section III is used. The selected reference model (in the required, by the algorithms, LFT structure) is the LFT-LPV 2nd order model described in Section IV.B. The dimension of its scheduling vector $\rho = \Delta_{Mach}$ ξ —containing the Mach dependence – is 20 and the dimension for the parametric and aerodynamic uncertainty operator $\eta = \Delta_{param \, ! \, aero}\xi$ 16. This 2nd order LPV_LFT was selected since its dimension was more amenable for validation of the methods and also since it can be considered a more crud, and thus conservative, model that the higher-order fitting models (i.e. if this model is not validated it is likely that the other models will not be validated either). The following structure characterizes the model:

$$\begin{aligned} \dot{x} &= M_{1,1}x + M_{1,2}\eta + M_{1,3}\rho + M_{1,4}u \\ \xi &= M_{2,1}x + M_{2,2}\eta + M_{2,3}\rho + M_{2,4}u \\ \zeta &= M_{3,1}x + M_{3,2}\eta + M_{3,3}\rho + M_{3,4}u \\ y &= M_{4,1}x + M_{4,2}\eta + M_{4,3}\rho + M_{4,4}u \end{aligned} \qquad \text{Eq. 17}$$

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Validation tests were performed using measurement data from the nonlinear simulation using the implementation from section III and *Figure IV-2*, at M=2.0 and Δ =0.2. The nominal model used is chosen with a reduced scheduling vector $\bar{\rho} = \Delta_{Mach}\bar{\zeta}$ of dimension 17 yielding the following structure:

$$
\begin{aligned}
\dot{x} &= M_{1,1}x + M_{1,2}\eta + \overline{M}_{1,3}\bar{\rho} + M_{1,4}u \\
\xi &= M_{2,1}x + M_{2,2}\eta + M_{2,3}\bar{\rho} + M_{2,4}u \\
\bar{\zeta} &= M_{3,1}x + \overline{M}_{3,3}\bar{\rho} + M_{3,4}u \\
y &= M_{4,1}x + M_{4,2}\eta + \overline{M}_{4,3}\bar{\rho} + M_{4,4}u
\end{aligned}
\qquad \text{Eq. 18}
$$

Five cases are tested with different values for the perturbation *w* and parametric uncertain set *Δ* (full block uncertainty and structured block-diagonal), see Table V-1. Besides verifying the developed toolbox, an additional aim of the selected tests is to investigate the sensitivity of the different implemented validation algorithms to the problem data and the conservatism introduced by the different assumptions used by each of the algorithms. During the preparation of the data special care is needed to obtain frequency domain data from time-domain measurement[§§§].

Table V-1 HL-20 model validation: test cases

Test Case	δ_d, noise gain range	Δ: type, bound
1	[0 2]	Full block, 0
2	[0 1]	Full block, 1
3	[0 2]	Full block, 0.2
4	[0 2]	Structured, 1
5	[0 2]	Structured, 0.2

The results validated the 2nd order LFT-LPV model for all the tests used. *Figure V-3* and *Figure V-4* show the frequency response magnitude obtained from the validation tests (should be less than 1 across frequency to indicate successful validation).
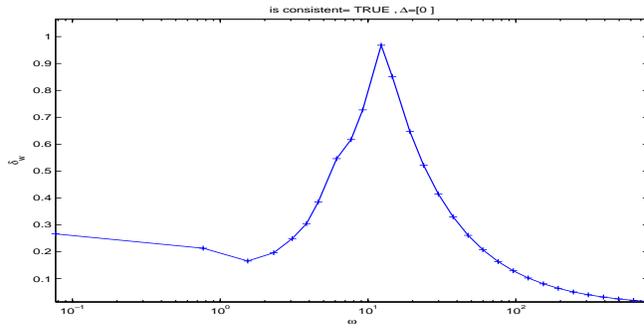

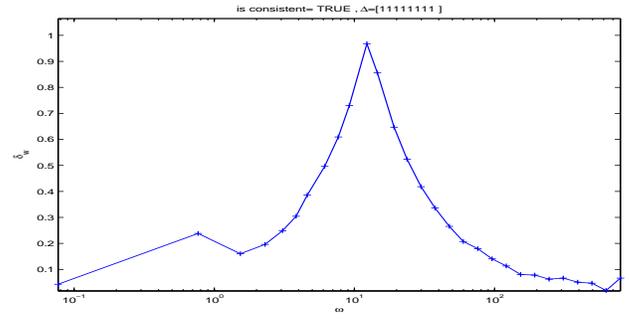**Figure V-3 Uncertainty validation: test case 2**

Figure V-4 Uncertainty validation: test case 4

As it was mentioned, the LPVMAD uncertainty validation tool can be used to facilitate a joint uncertainty modeling and controller synthesis process. Thus, this capability was demonstrated by finding a reduced-order weight that can explain the uncertainty present in the system for a set of data runs. The result is shown in *Figure V-5* where the initial and final performance level γ are shown together with those from the overbounding reduced-order weight. This capability is especially important for the future impact of this tool in the control design cycle, as it will help finding and validating weights for design given an LFT model and time/frequency domain data from the real systems.

[§§§] It is noted that the model validation is performed against the Simulink/Matlab implementation of the nonlinear system, thus the data used is not 'real' flight data. Nevertheless, the implemented model can be reasonably taken as the 'real' system as the data it provides contain the main features of the system in terms of nonlinearities and noise (it is also noted that for Space systems the availability of 'real' flight data is challenging).
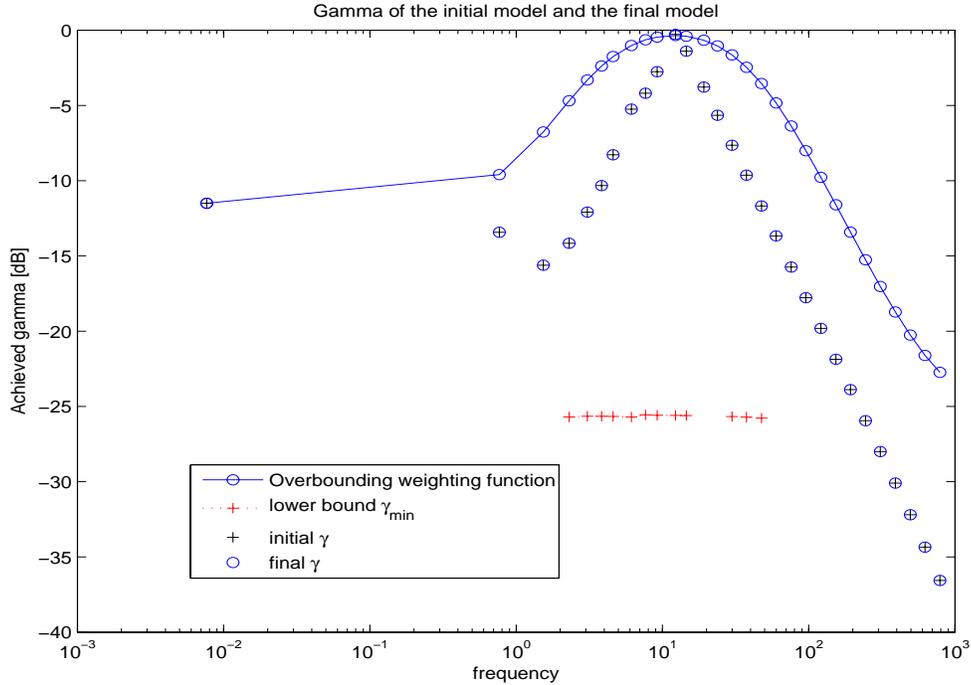
Figure V-5 Uncertainty validation: weight estimation

In conclusion, the results from the assessment of the LPVMAD data-based model validation tool show that no model invalidation occurs for the selected 2nd order LFT-LPV model. However, the sufficient conditions are quite conservative, i.e. allows only a slightly reduction of the accepted noise level δ_d to tune efficiently the corresponding weighting functions. The general nonlinear optimization algorithm used passed with a much lower γ value, which bounds both the uncertainty and the noise, but at the expense of an increased computational demand. The tests showed that further efforts are needed in order to design numerically efficient algorithms that are able to use the available measurement data for model validation and to facilitate the control oriented uncertainty modeling by providing the best weighting functions.

VI.   Conclusion

In this article an assessment of the advantages and disadvantages of LPV/LFT modeling and data-based validation technology for their use in the control design process for space system has been presented. The assessment is based on the development of corresponding tools and their application to a space benchmark, the longitudinal nonlinear motion of NASA's HL-20 re-entry vehicle. The results show that techniques and tools based on automated LFT/LPV modeling (combining numerical linearization and symbolic manipulation approaches) are sufficiently mature to be introduced in the control design process within an LPV framework –albeit still requiring further research to assess issues such as trim effects on the LFT. The conclusion from the assessment also indicates that LPV data-based model validation approaches are not yet at a mature stage to be used within an industrial design process due to the mathematically complex underpinning of the approaches and the limited availability of numerically reliable tools and algorithms. Nevertheless, the results from the application of these model-validation techniques to the selected problem represent a preliminary but successful validation of the potential for further evolution of these techniques and tools towards their introduction within the (LPVMAD) control design framework [34].

Acknowledgments

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