ROBUSTIFICATION OF STATIC AND LOW ORDER ANTI-WINDUP DESIGNS

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Abstract:
In this paper an approach for improving the robustness of static and low order anti-windup compensators is introduced. These types of anti-windup compensators are typically designed to recover performance in the face of windup but can result in loss of performance (including stability) for the closed loop in the face of uncertainty. The proposed approach improves the anti-windup design by incorporating an additional robustifying compensator which is active when uncertainty is present in the system. An appealing practical feature of the scheme is that, for the static anti-windup case at least, the number of additional states is kept to a minimum.

Keywords: Anti-windup, Robust control design

1. INTRODUCTION

Actuator saturation is a long-standing problem faced by control engineers. A popular approach for coping with it is to use anti-windup (AW) techniques. In such an approach, the assumption is that a satisfactory, stabilising, baseline (normally linear) controller has been designed for the plant under consideration, in the absence of actuator saturation. The task of the AW compensator, which is only activated under saturation, is then to limit performance degradation during this period and to ensure a graceful return to linear behaviour (Kothare, M.V. *et al.*, 1994).

However, most AW schemes tackle the problem of actuator saturation from a *nominal performance and stability* perspective whereby it is assumed that the anti-windup design inherits the robustness properties of the (hopefully robust) linear system (Kothare, M.V. *et al.*, 1994; Weston, P.F. and Postlethwaite, I., 2000). This makes intuitive sense although it might be argued that robustness of the linear system is only a necessary, but not a sufficient, condition for robustness of the entire saturated closed loop (Turner, M.C. *et al.*, 2004). Indeed, in the latter reference it was shown that a ‘good’ static AW design for the nominal case actually destabilised the saturated closed-loop in the presence of uncertainty.

The aim of this paper is to present a simple approach to the robustification of AW compensators which have been designed with a nominal performance-objective goal (e.g. quadratic stabilizability of nominal closed-loop). The scheme is based on the theory of residual generators and has the potential to improve the performance and stability objectives of the unsaturated and saturated closed loops.

2. ANTI-WINDUP PROBLEM DEFINITION

The objective of AW compensation is rather subjective, although it typically (Kothare, M.V. *et al.*, 1994; Weston, P.F. and Postlethwaite, I., 2000; Zaccarian, L. and Teel, A., 2002) involves the modification of a

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baseline (linear) controller – with good performance and robustness in the unconstrained case– so that:

1. The desired closed-loop performance and robustness are achieved when the initial conditions and reference signals do not cause saturation.
2. In the event of saturation, the constrained response is close to the unconstrained behaviour (although obviously limited by infeasible demands).
3. The recovery of the desired characteristics after saturation is fast.

A scheme which represents most AW configurations is given in (Weston, P.F. and Postlethwaite, I., 2000), where an interpretation of the conditioning of nominal controllers $K(s)$ in terms of a single transfer function $M(s)$ is given. The set-up for the Weston-Postlethwaite anti-windup (WP-AW) approach is depicted in Figure 1. The generalized plant is given by $G(s) = [G_d(s) \;\; G_o(s)] \in \mathcal{RH}_\infty$ (assumed exponentially stable) and the controller by $K(s) = [K_w(s) \;\; K_o(s)]$. The signal $w \in \mathbb{R}^{n_w}$ is the reference, $d \in \mathbb{R}^{n_d}$ is the disturbance on the plant and $y \in \mathbb{R}^p$ is the plant output. The control signal is $u \in \mathbb{R}^m$ and the actual plant input is $\text{sat}(u) = u_m \in \mathbb{R}^m$. The signals $u_d \in \mathbb{R}^m$ and $y_d \in \mathbb{R}^p$ are produced by the anti-windup compensator $[(M-I)' \;\; (G_oM)']'$.

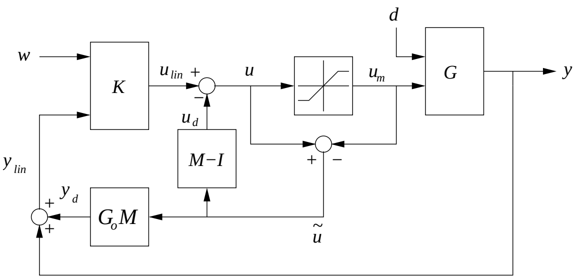

Fig. 1. Weston-Postlethwaite AW scheme.

The saturation and deadzone functions are defined as

$$\text{sat}(u) = \begin{bmatrix} \text{sat}_1(u_1) \\ \vdots \\ \text{sat}_m(u_m) \end{bmatrix} \quad \text{Dz}(u) = \begin{bmatrix} \text{Dz}_1(u_1) \\ \vdots \\ \text{Dz}_m(u_m) \end{bmatrix} \quad (1)$$

and are related through the identity

$$\text{sat}_i(u_i) = u_i - \text{Dz}_i(u_i) \quad (2)$$

where for all $i \in \{1, \ldots, m\}$ the saturation nonlinearity is given by $\text{sat}_i(u_i) = \text{sign}(u_i)\min(|u_i|, \overline{u_i})$; the deadzone by $\text{Dz}_i(u_i) = \text{sign}(u_i)\max(0, |u_i| - \overline{u_i})$; and $\overline{u_i} > 0$ is a fixed bound. Using the above, it can be shown that Figure 1 can be re-drawn as Figure 2 which reveals an attractive decoupling into a nominal linear system, a nonlinear loop, and a disturbance filter. Note that if no saturation occurs ($\tilde{u} = 0$), then the nominal linear system is all that is required to determine the system's behaviour. However, if saturation occurs ($\tilde{u} \neq 0$), the nonlinear loop and disturbance filter become active.

Using this representation, the question of global stability for the complete system is translated into determining whether the nonlinear loop is stable. Also, note that the dynamics of the disturbance filter determine the manner in which the nominal linear behaviour is affected during and after saturation.

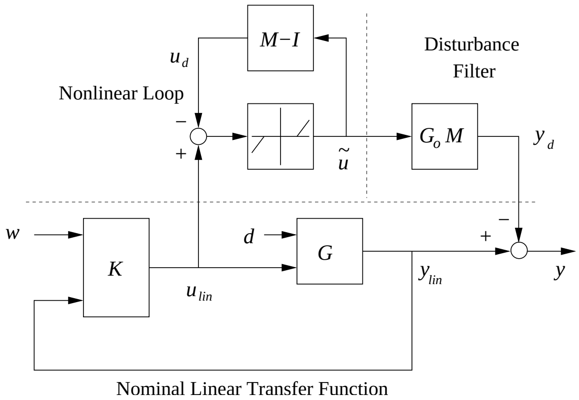

Fig. 2. Equivalent representation of WP-AW.

3. NOMINAL STATIC AND LOW-ORDER ANTI-WINDUP SYNTHESIS

The main appeal of a static AW compensator is its simplicity and ease of implementation. Given a plant, G_o, with state dimension n_p, the full-order compensators of (Grimm, G. *et al.*, 2003) and (Turner, M.C. *et al.*, 2004) add an extra n_p states to the controller in order to solve the AW problem. The static AW compensator, by definition, adds no extra states. Low order compensators, see (Turner, M.C. and Postlethwaite, I., 2004), have similar appeal.

The framework of the previous section is useful for the synthesis of static and low order AW compensators, (Turner, M.C. and Postlethwaite, I., 2004). The implementation is shown in Figure 3 where the AW compensator, $\Theta(s) = [\Theta_1(s)' \quad \Theta_2(s)']'$, emits two signals, $\theta_1 \in \mathbb{R}^m$ and $\theta_2 \in \mathbb{R}^p$, similar to the scheme in Figure 1. More generality of the scheme can be obtained by injecting θ_2 directly into the controller state equation instead of at the controller input, as suggested in (Grimm, G. *et al.*, 2003; Kapoor, N. *et al.*, 1998), although this may not always be possible in practical problems. However, these results can be recovered in the framework adopted here by appropriately augmenting the controller input distribution matrix with extra columns and the output y with extra null entries.

From Figures 1 and 3, respectively, it follows that

$$u = K_w w + K_o(y + G_o M\tilde{u}) - (M-I)\tilde{u} \quad (3)$$
$$u = K_w w + K_o(y + \Theta_2\tilde{u}) - \Theta_1\tilde{u} \quad (4)$$

Thus, $M(s)$ can be written as

$$M = (I - K_o G_o)^{-1}(-K_o\Theta_2 + \Theta_1 + I) \quad (5)$$

If we restrict our attention to static Θ, using this expression for $M(s)$ it is then possible to derive LMI conditions which allow the synthesis of a static compensator Θ guaranteeing the stability of the closed-loop system and minimising an upper bound on the $\mathcal{L}_2$ gain of the map $\mathcal{T}_p : u_{lin} \mapsto y_d$ (Turner, M.C. and Postlethwaite, I., 2004). Note that the mapping $\mathcal{T}_p$ is central to the AW performance objectives as this operator represents the deviation of the saturated behaviour of the system from its unconstrained behaviour (see Figure 2 for a clearer understanding). If the induced norm of this operator is small, intuitively, we can expect good anti-windup performance.

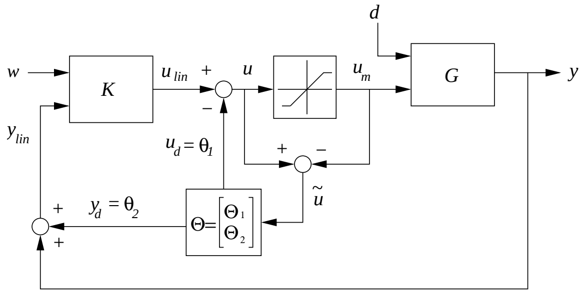

Fig. 3. Structure of generic AW scheme.

4. UNCERTAINTY IN ANTI-WINDUP SCHEMES

Unfortunately, as stressed in (Turner, M.C. and Postlethwaite, I., 2004), a serious weakness of almost all static and low order AW schemes found in the literature is that plant uncertainty is neglected in the design of the AW compensator. This is a standard assumption in most AW papers since it is not trivial to incorporate robustness directly into their design. Some steps in this direction are described in (Garcia, G. *et al.*, 1999; Turner, M.C. *et al.*, 2004; Galeani, S. *et al.*, 2005).

In order to see the difficulty which arises from uncertainty considerations, consider the true plant is given by $\tilde{G}(s) = [G_d(s) \;\; G_o(s) + \Delta_G(s)] \in \mathcal{RH}_\infty$ (where we are assuming additive uncertainty since it is one of the most general and widely used representations for uncertainty). Starting from Figure 1 with $G(s)$ replaced by $\tilde{G}(s)$ and following the loop manipulations from (Turner, M.C. *et al.*, 2004), it is easy to re-draw that configuration as in Figure 4.

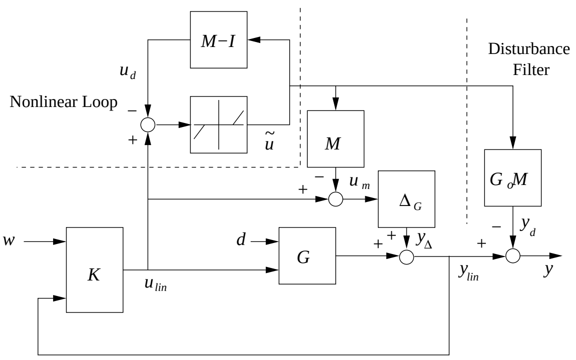

Fig. 4. Uncertain representation of WP-AW.

Note that the term $\Delta_G M : \tilde{u} \mapsto y_\Delta$ destroys the decoupling of the linear and nonlinear loops. This loss of decoupling requires several additional assumptions in order to obtain global stability results, mainly that the uncertainty $\Delta_G(s)$ is asymptotically stable and small in some sense. This in turn implies a Small-Gain argument for the stability of the saturated system which reduces its general applicability (to systems within the family obtained from the reduced uncertainty).

A further problem exists with static AW schemes: they are known to stabilise quadratically only a subset of plant-controller combinations in the presence of saturation (Grimm, G. *et al.*, 2003). This means that if a static AW compensator exists for the nominal plant, G, there is no guarantee it would exist for the perturbed plant $\tilde{G}(s)$.

5. ROBUSTIFICATION OF STATIC AND LOW-ORDER AW SCHEMES

In this section, a practical and simple approach is proposed for the robustification of AW compensators. The idea is to augment the WP-AW configuration with a residual generator which is active when uncertainty is present – hence, it can be interpreted as a robustifying controller for the unsaturated and saturated loops.

The concept of a residual generator is well-known in the field of fault detection and isolation (FDI) where it is used to detect and isolate faults in a monitored system using its input and output signals. A general frequency residual generator can be obtained using the left coprime factorization of the nominal plant $G_o(s) = \tilde{M}^{-1}(s)\tilde{N}_o(s)$ (Ding, X. and Frank, P.M., 1990):

$$r = Q\tilde{r} = Q(\tilde{N}_o u - \tilde{M} y) \qquad (6)$$

Consider Figure 5 and assume the plant is given by the true system $\tilde{G}(s)$ (with additive uncertainty and no faults or disturbances for simplicity of presentation), then the residual signal $r(s)$ becomes:

$$\begin{aligned} r =& Q(\tilde{N}_o u - \tilde{M} y) = Q(\tilde{N}_o u - \tilde{N}_o u - \tilde{M}\Delta_G u) \\ =& -Q\tilde{M}\Delta_G u \end{aligned} \qquad (7)$$

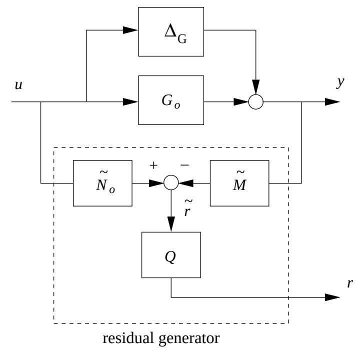

Fig. 5. General residual generator.

In this fashion, the primary residual $\tilde{r} = \tilde{M}\Delta_G u$ can be interpreted as an uncertainty monitor: it is equal to zero for the nominal plant where $\Delta_G(s) = 0$, and for the true plant case it equals the filtered uncertainty. The assumption of no disturbances is not restrictive since if they were to be present, they would also appear in the residual signal but then the filter $Q(s)$ would be designed to reject/minimize their effects on the residual.

Using this concept it is straightforward to robustify the WP-AW scheme by augmenting its general configuration, Figure 3, with a coprime residual generator using the plant output $y(s)$ and saturated input $u_m(s)$, see Figure 6.

From direct observation of the configuration in Figure 6 it is clear how the robustified version of the WP-AW scheme operates in order to achieve its objectives in the presence of saturation and/or uncertainty:

Case 1. No uncertainty ($\tilde{r} = 0$), no saturation ($\tilde{u} = 0$). The nominal linear loop (G_o, K) retains the performance and stability objectives for which it was designed.

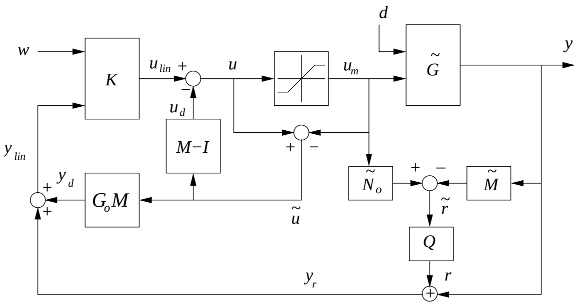

Fig. 6. Robustified general WP-AW scheme.

Case 2. No uncertainty ($\tilde{r}=0$), saturation ($\tilde{u}\neq 0$). The configuration from Figure 1 is obtained – with the aforementioned advantages for anti-windup synthesis and analysis.

Case 3. Uncertainty ($\tilde{r}\neq 0$), no saturation ($\tilde{u}=0$). The filter $Q\tilde{M}$ becomes a robustifying controller that minimizes the uncertainty effects on the nominal loop (G_o, K), see Figure 7.

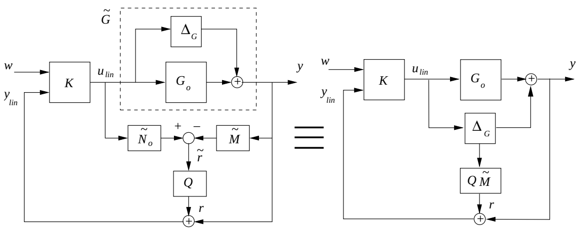

Fig. 7. Robustifying control.

Case 4. Uncertainty ($\tilde{r}\neq 0$) and saturation ($\tilde{u}\neq 0$). Similar to the previous case, the filter $Q\tilde{M}$ represents an additional degree-of-freedom to robustify the saturated closed-loop.

Notice that from these observations the idea that the augmented WP-AW configuration represents a general scheme for robust AW is convincing (Marcos, A. *et al.*, 2006). Manipulating the input-output maps in the spirit of the manipulations required to obtain Figure 2, it is possible to obtain the scheme shown in Figure 8.

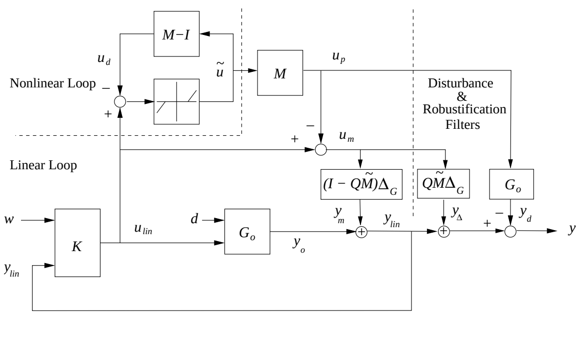

Fig. 8. Representation of the general WP-AW scheme for analysis.

This analytical representation provides further insight on the interactions and roles of the different design parameters in the configuration, i.e. Q, M, $\tilde{M}$ and $\tilde{N}_o$, and provides guidelines for their design. Specifically, the configuration can be divided into the same three sections from Figure 2: linear loop, nonlinear loop, and disturbance (now augmented with robustification) filters. It is observed that the linear and nonlinear loops are still coupled although this time through the term $y_m = (I - Q\tilde{M})\Delta_G u_m$. Also, there is a further coupling on the outer-loop $y = y_{lin} + y_\Delta - y_d$ due to the robustifying filter $y_\Delta = Q\tilde{M}\Delta_G u_m$. A particularly large amount of design freedom lies in the free parameter, $Q(s)$, and there are two particularly striking choices:

(1) $Q = 0$. In this case the results of (Turner, M.C. *et al.*, 2004) and thus Figure 4 are recovered; the coupling with the outer-loop is lost while the coupling between the linear and nonlinear loop occurs due to the uncertainty Δ_G. This implies that nominal stability is affected by the presence of uncertainty and saturation, and therefore the design of AW compensators need to account for this coupling. Furthermore, global stability conditions are limited by the Small-Gain Theorem – see Section 4. This is the set-up used recently in a robust anti-windup LMI approach (Turner, M.C. *et al.*, 2004), which tries to optimise the stability robustness by minimizing the $\mathcal{L}_2$ gain of the map $\mathcal{T}_r : u_{lin} \mapsto u_m$. Interestingly, it is noted in that reference that the robustness of the linear system can never be improved in this case – which is in agreement with the results here.

(2) $Q = \tilde{M}^{-1}$. In this case a complete decoupling of the linear and nonlinear loops is obtained. Indeed, this case represents a loop shifting of the uncertainty coupling from the inner linear loop to the outer-loop. This has obvious advantages as in this case a Small-Gain condition for global stability of the saturated system is *not necessary* (although it is desirable for robust performance). Moreover, this implies that minimisation of the map $\mathcal{T}_p : u_{lin} \mapsto y_d$ (as in (Turner, M.C. and Postlethwaite, I., 2004)) would automatically result in a stable system, regardless of the uncertainty present.

In summary, choosing $Q = \tilde{M}^{-1}$ implies essentially unconditional robust stability at the expense of robust performance; $Q = 0$, implies the potential of nominal robustness recovery during saturation and nominal linear behaviour in the absence of saturation. Thus, from Figure 8, it can be seen that $M(s)$ can be designed to tackle normal AW performance and stability goals, i.e. minimising the map $\mathcal{T}_p : u_{lin} \rightarrow y_d$; and the choice of Q can be used to trade-off robust stability and robust performance (of both the unsaturated and saturated closed-loop) in a clear manner.

One aspect of the scheme which should be mentioned is that, in contrast to normal anti-windup schemes, the presence of the residual generator means that $Q(s)$ is *always active*, providing uncertainty/disturbances are present. This relates our approach to the so-called weakened AW approach (Galeani, S. *et al.*, 2005) developed on a relaxation of the uncertainty and a filter $F \equiv I - Q$ which directly minimizes the effect of the uncertainty on the control signal u_{lin}.

It is noted that the proposed robustification approach is especially practical for static or low order AW approaches (Turner, M.C. and Postlethwaite, I., 2004) due to the increase in the number of states – arising from $\tilde{N}_o$, $\tilde{M}$, Q in the residual generator. Indeed, choosing Q static, means no more states are required than a standard full-order AW compensator.

From the previous guidelines and comments, the following algorithm is proposed for the robust design of static and low order AW designs:

Step 1. Design a two Degrees-of-Freedom (DoF) controller K to satisfy the required performance and stability objectives for the nominal plant G_o under no saturation.

Step 2. Design a static or low order AW compensator to satisfy the "true goal" of AW design: minimization of the deviation between the behaviour of the nonlinear loop and that of the linear loop during and after saturation. This AW criterion can be formulated as in (Turner, M.C. and Postlethwaite, I., 2004).

Step 3. Design Q (and/or coprime factors $\tilde{M}, \tilde{N}_o$) so that the nominal performance and stability objectives for the unsaturated and saturated cases are minimally degraded in the presence of uncertainty, see (Marcos, A. *et al.*, 2006) for more design details.

Remark 1. In Step 1 a high-performance design objective has been defined (i.e. control of the nominal plant) since the architecture proposed allows the robustness characteristics to be accomplished through the design of Q in Step 3. More typically, the controller in Step 1 will be designed as well for robustness purposes of the unsaturated closed loop (i.e. control of the nominal and uncertain plants) and Q will then provide the robustification of the saturated loop.

Remark 2. This paper, as is standard in the AW literature, considers *global* asymptotic stability of the resulting AW system. Therefore, it is necessary to consider exponentially stable nominal and uncertain plants, and thus the implementation of the residual generator in Step 3 can be given by $\tilde{r} = Q(G_o u_m - y)$ which implies that the design effort can focus on Q.

6. MASS-SPRING-DAMPER EXAMPLE

The nominal and perturbed mass-spring-damper example from references (Zaccarian, L. and Teel, A., 2002; Turner, M.C. *et al.*, 2004) is used to demonstrate the robustification approach. There is no restriction to SISO systems in the proposed approach but this SISO example facilitates its understanding. The state-space of the nominal plant $G_o(s)$ is given by:

$$A_o = \begin{bmatrix} 0 & 1 \\ -10 & -10 \end{bmatrix}; B_o = \begin{bmatrix} 0 \\ 10 \end{bmatrix}; C_o = [1 \ 0]; D_o = [0]; \quad (8)$$

A two-degrees-of-freedom linear controller with acceptable performance (fast convergent response, no steady-state error, no overshoot) and robustness (asymptotic tracking and rejection of step disturbances) is given in the same two references. Its state-space is $K(s) = [K_w(s) \ K_o(s)] \sim (A_c, [B_{cr} \quad B_c], C_c, [D_{cr} \quad D_c])$ where:

$$A_c = \begin{bmatrix} -80 & 0 & 2.5 \\ 1 & 0 & 0 \\ 0 & 0 & -2.5 \end{bmatrix}; B_{cr} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}; B_c = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix};$$

$$C_c = [-9450 \ 3375 \ 337.5]; D_{cr} = [0]; D_{cy} = [-135];$$

In (Turner, M.C. *et al.*, 2004), a quadratically stabilizable static anti-windup compensator is designed for the nominal system. From simulation it is known that this static AW design is not stable for the perturbed system (shown later). The implementation of the resulting static AW gains $\Theta = [-0.1909 \quad 0.1402]^\top$ is the same as that in Figure 3 with $G(s) = G_o(s)$.

The nominal plant output responses are shown in Figure 9. The thick solid and dashed-dot lines are the responses of the nominal, with no AW, closed-loop system under the unconstrained and the constrained (plant input saturation limited to ± 1 meters) environments, respectively. It is observed that for the input saturation case, the response performance has degraded and it is out-of-phase with respect to the unsaturated response. The thick dashed line in the figure shows the response of the saturated closed-loop with the static AW implementation of Figure 3. It shows performance improvement since the response is now in-phase with the reference command (although the infeasibility of the command prevents correct steady-state tracking beyond the saturation magnitude).

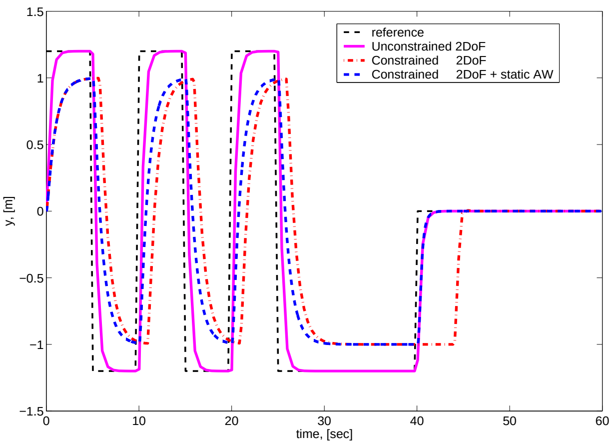

Fig. 9. Responses of the nominal closed-loops.

Next, we examine the case of the true, or perturbed, plant $\tilde{G}(s) = G_o(s) + \Delta_G(s)$ which has a large resonant peak. Its state-space representation is given by:

$$A_p = \begin{bmatrix} 0 & 1 \\ -10 & -0.01 \end{bmatrix}; B_p = B_o; C_p = C_o; D_p = D_o; \quad (9)$$

Note that (9) is a large perturbation away from (8): the open-loop poles change from $(-8.87, -1.13)$ to $-0.05 \pm 3.16j$. Figure 10 shows the same responses as before but now using the perturbed plant. It is noted that the perturbed response of the constrained linear 2 DoF controller without AW (the thick dashed-dot line) is robust to the uncertainty and only results in an oscillatory behaviour during transients in comparison to its nominal response from Figure 9 (it still contains the out-of-phase characteristic). On the other hand, it is clear that the static AW (thin dashed line) not only has not inherited the robustness properties of the linear closed loop but it has made it unstable.

Using the approach from the previous section, a robustifying controller is obtained to address the lack of robustness for the static AW. The controller is a first order low-pass filter obtained after a few trial-and-error selections. The implementation of this ad-hoc robustifying controller is shown in Figure 11 where the low-pass filter is given by:

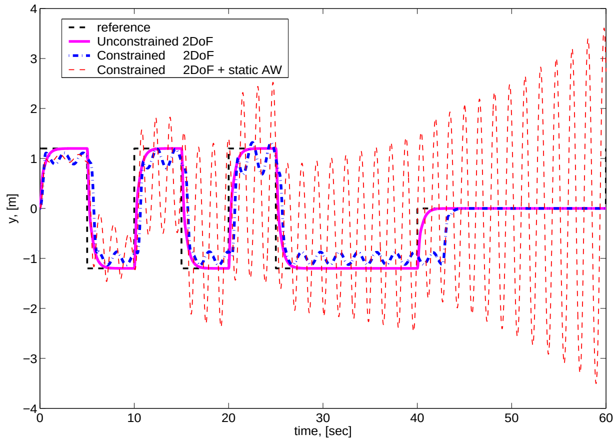

Fig. 10. Responses of the perturbed closed-loop.

$$Q(s) = \frac{1.7}{s+10} \tag{10}$$

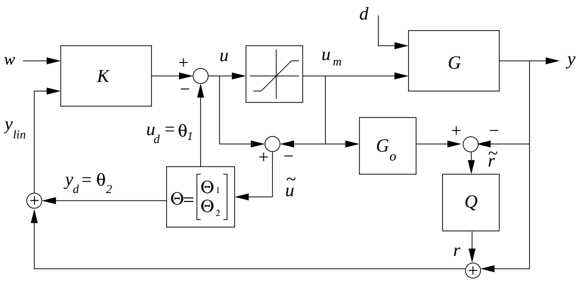

Fig. 11. Structure of robustified static AW scheme.

As mentioned in Section 5, the residual signal $r = G_o u_m - y$ is identically to zero when no uncertainty (or disturbances) are considered in the system. Therefore, the time responses for the nominal plant are exactly as those given in Figure 9. When the true system $\tilde{G}(s)$ is used in conjunction with the linear controller plus the static AW and the residual generator, – i.e. the scheme from Figure 11 – the perturbed time responses are given by Figure 12.

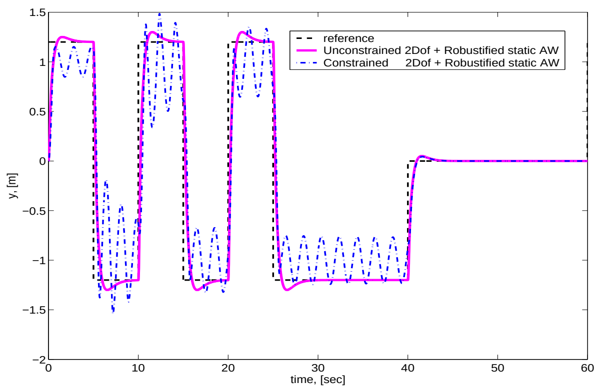

Fig. 12. Responses of the robustified static AW perturbed closed-loop.

Observe that there is a slight loss of performance (i.e. small overshoot) for the unconstrained response (thick solid line) arising from the contribution of the filtered uncertainty. For the saturated case (thick dashed-dot line), it is clear that the new scheme has succeeded in robustifying the static anti-windup since the response is now stable albeit oscillatory during saturation. Indeed, the response is similar to the perturbed, constrained response for the 2 DoF linear controller without AW, see Figure 10, except that the out-of-phase behaviour is corrected. This seems to indicate a need to re-design the baseline controller rather than spend more time optimizing the anti-windup or the robustifying controller.

7. CONCLUSIONS

In this paper a simple extension of the Weston-Postlethwaite approach for robust AW design has been presented. The robustification of the WP-AW scheme depends on the inclusion of a residual generator which acts as an uncertainty or disturbance monitor and can be designed to be a robustifying controller when uncertainty is present. A simple mass-spring-damper example shows the potential of the approach to robustify a static AW design which otherwise might destabilise the saturated, perturbed closed-loop system.

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