LINEAR FRACTIONAL TRANSFORMATION FORMULATION OF THE INTEGRATED CONTROLLER APPROACH

Andrés Marcos[†], Gary J. Balas[‡]

[†] Post-Doctoral Fellow, Department of Engineering, Control & Instrumentation Group,
University of Leicester, University Road, Leicester, LE2 2LG, UK
fax: +44 0116 e-mail: ame12@le.ac.uk
[‡] Professor, Department of Aerospace Engineering and Mechanics, University of Minnesota,
107 Akerman Hall, 110 Union St. SE, Minneapolis
e-mail: balas@aem.umn.edu

Keywords: LFT, integrated controller, fault diagnosis, aircraft.

Abstract

In this paper a general linear fractional transformation (LFT) formulation of the four-parameter controller is provided. The LFT formulation provides a more transparent look at the interactions between the control and the diagnostic objectives. This general LFT is then particularized to a special architecture widely used within the fault tolerant/diagnostic community.

1 Introduction

The design of active fault tolerant and reliable controllers have benefited from the development of the FDI field and the possibility of accurately identifying faults on a system. This has resulted in a new trade-off which arises between the controller and the FDI filter objectives. This trade-off has been analyzed in references [4, 5, 11] where it was shown that the conflict stems from coupling of the controller and filter objectives when uncertainty is present.

In order to address this trade-off a number of approaches have been proposed in the literature. In reference [4] an integrated control/filter approach based on the Youla parameterization was proposed. Solutions based on optimal and robust theory were proposed for such approach in [9]. Reference [5] particularized the controller to two degrees-of-freedom (feedback control/diagnostic signal generation) and studied the problem, using the standard robust control configuration, for nominal and uncertain systems. In reference [11], a mixed $\mathcal{H}_2/\mathcal{H}_\infty$ criterion was used to design a reconfigurable control with optimized control and diagnostic performance indexes.

With a different motivation, i.e. high-performance controller architectures, the Youla and Dual Youla parameterizations were studied in [8]. This reference showed that there is a separation principle for this type of controllers (connected with the residual generation theory). Recently, a breadth of papers have proposed similar architectures based on the high performance architecture and the integrated approach, see [7, 6, 13].

The main goal of this paper is to derive general stability conditions and formulae for the linear fractional transformation formulation of the four-parameter controller. Furthermore, the afore mention architectures for the integrated approach are also studied and derived from this general LFT formulation.

2 General Four-Parameter Controller

The four-parameter controller is a generalization of the Youla parameterization which extends the controller to four degrees of freedom (DoF) by considering reference signal tracking, closed-loop stabilization, residual generation, and disturbance rejection. Figure 1 shows the structure for the parameterization of all four-parameter controllers given a system $G_u = NM^{-1} \in \mathcal{R}_P$, the general result is formalized in Theorem 2.1. A feedforward controller, $K_{ff} = \tilde{V}^{-1}\tilde{U}_{ff}$, is included in this architecture. The coprime factor $\tilde{V}$ is assumed common with feedback controller K_o due to the well-known requirement that K_{ff} be stable or if unstable, implemented together with K_o.

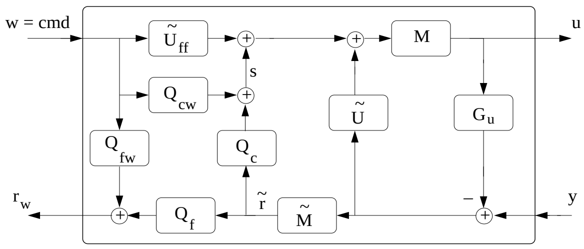

Figure 1: Four-parameter controller parameterization structure.

Theorem 2.1 (General Four-parameter Controller)
Consider a nominal plant $G_u \in \mathcal{R}_P$. Assume a corresponding nominal stabilizing controller, $K_o \in \mathcal{R}_P$, and feedforward controller, $K_{ff} \in \mathcal{R}_P$, are given. Let any right/ left coprime factorization (r.c.f. / l.c.f.) for the nominal plant, $G_u = N_u M^{-1} = \tilde{M}^{-1}\tilde{N}_u$, and the controllers, $K_o = UV^{-1} = \tilde{V}^{-1}\tilde{U}$; $K_{ff} = U_{ff}V^{-1} = \tilde{V}^{-1}\tilde{U}_{ff}$, be known. The class of all proper integrated (stabilizing and residual generator) controllers $K_F(Q) \in \mathcal{R}_P$ is parameterized by:

$$\begin{bmatrix} u \\ r_w \end{bmatrix} = \begin{bmatrix} (\tilde{V} + Q_c\tilde{N}_u)^{-1}(\tilde{U} + Q_c\tilde{M}) & (\tilde{V} + Q_c\tilde{N}_u)^{-1}(\tilde{U}_{ff} + Q_{cw}) \\ Q_f\left(\tilde{M} - \tilde{N}_u(\tilde{V} + Q_c\tilde{N}_u)^{-1}(\tilde{U} + Q_c\tilde{M})\right) & Q_{fw} - Q_f\tilde{N}_u(\tilde{V} + Q_c\tilde{N}_u)^{-1}(\tilde{U}_{ff} + Q_{cw}) \end{bmatrix} \begin{bmatrix} y \\ w \end{bmatrix} \quad (1)$$

Under the following conditions: $(\tilde{V}+Q_c\tilde{N}_u)(\infty)$ *exist and*

$$\begin{bmatrix} s \\ r_w \end{bmatrix} = Q \begin{bmatrix} \tilde{r} \\ w \end{bmatrix} = \begin{bmatrix} Q_c & Q_{cw} \\ Q_f & Q_{fw} \end{bmatrix} \begin{bmatrix} \tilde{r} \\ w \end{bmatrix} \in \mathcal{RH}_\infty, \quad (2)$$

where u *is the feedback control input,* r_w *the residual vector,* y *the plant measurements and* $w = cmd$ *the exogenous input.*

Proof: Based on Theorem (33) in [4] and Theorem 12.17 from [12], see reference [3].

Remark 1. The controller parameterization of equation (1) is an adaptation, but equivalent after setting $\tilde{U}_{ff} = 0$, to the formulae given in [4]. As it is observed in Figure 1, the class of four-parameter controllers is associated with the implicit internal model principle [13]. This association is highlighted by the internal signal, the primary residual, $\tilde{r} = \tilde{M}y - \tilde{N}_u u$ which is connected with the residual generation theory [2]: the parameterization of all residual generators for a system $G = G_u u + G_f f + G_d d$ can be given by:

$$r = Q_f \tilde{r} = Q_f(\tilde{M}y - \tilde{N}_u u) = Q_f(\tilde{N}_f f + \tilde{N}_d d) \quad (3)$$

Remark 2. Removing the second row in the controller, corresponding to the diagnostic signal generation, the standard two degree-of-freedom controller parameterization is obtained [10]. Furthermore, the controller term K_{11} corresponds to the well-known Youla parameterization. An integrated two DoF design (i.e stabilizing controller with diagnostic signal generation) can be obtained assuming no contribution from the commands, i.e. $cmd = 0$:

$$\begin{bmatrix} K_1(Q_c) \\ K_2(Q_c, Q_f) \end{bmatrix} = \begin{bmatrix} K_o + \tilde{V}^{-1}Q_c(I + V^{-1}N_uQ_c)^{-1}V^{-1} \\ Q_f(I + V^{-1}N_uQ_c)^{-1}V^{-1} \end{bmatrix} \quad (4)$$

If the plant is stable (i.e. $\tilde{V} = V = I; \tilde{U} = U = 0; \tilde{N}_u = N_u = G_u; \tilde{M} = M = I$) the integrated two-parameter controller given in reference [5] is obtained. That reference also showed that there is a separation principle for the nominal case that is not present for the uncertain case. This leads to the conclusion that for the uncertain case it is more advantageous to design the controller and the fault detection filter within an integrated framework to better trade-off the filter performance and the controller robustness.

3 LFT Formulation Four-Parameter Controller

A general stability result for a generalized plant P, given by equation (5), and the four-parameter controller $K_F(Q)$, equation (1), is derived in this section together with general formulae for the controller formulation as a linear fractional transformation. For clarity purposes, the general plant and four-parameter controller are shown in compact notation:

$$\begin{bmatrix} e \\ y \end{bmatrix} = P \begin{bmatrix} w \\ u \end{bmatrix} = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \begin{bmatrix} w \\ u \end{bmatrix} \quad (5)$$

$$\begin{bmatrix} u \\ r \end{bmatrix} = K_F(Q) \begin{bmatrix} y \\ cmd \end{bmatrix} = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{bmatrix} y \\ cmd \end{bmatrix} \quad (6)$$

Note that now the diagnostic signal is denoted by r and it is equivalent to r_w in equation (1). Also, possibly different exogenous signals are assumed for the plant and the controller, i.e. w and cmd.

Theorem 3.1 (Stability Conditions for $(\mathbf{P}, \mathbf{K_F}(\mathbf{Q}))$.) *Given a generalized plant* $P \in \mathcal{R}_P$ *(with* $P_{22} = G_u \in \mathcal{R}_P$*), and the general four-parameter controller* $K_F(Q) \in \mathcal{R}_P$*. Necessary and sufficient conditions for* $K_F(Q)$ *to stabilize* P *are given by:*

$$\begin{bmatrix} I & -K_{11} \\ -P_{22} & I \end{bmatrix}^{-1} \in \mathcal{RH}_\infty; \quad (7)$$

$$P \begin{bmatrix} I \\ (I - K_{11}P_{22})^{-1}K_{11}P_{21} \end{bmatrix} \in \mathcal{RH}_\infty \quad (8)$$

$$\begin{bmatrix} P_{12} & P_{21} \end{bmatrix} (I - K_{11}P_{22})^{-1} \begin{bmatrix} I & K_{12} \end{bmatrix} \in \mathcal{RH}_\infty \quad (9)$$

$$\begin{bmatrix} I & P_{12} \\ 0 & P_{22} \end{bmatrix} (I - K_{11}P_{22})^{-1}K_{12} \in \mathcal{RH}_\infty \quad (10)$$

Proof: Immediate proof by applying the internal stability test:

$$\begin{bmatrix} I & -\begin{bmatrix} 0 & 0 \\ 0 & K_F(Q) \end{bmatrix} \\ -\mathcal{P}^a & I \end{bmatrix}^{-1} \in \mathcal{RH}_\infty \quad (11)$$

where the generalized plant P has been augmented to account for the controller exogenous inputs and outputs, i.e. r and cmd. Using compact notation: $\tilde{e} = [e^\top \ r^\top]^\top$, $\tilde{y} = [y^\top \ cmd^\top]^\top$, $\tilde{w} = [w^\top \ cmd^\top]^\top$ and $\tilde{u} = [u^\top \ r^\top]^\top$, the following augmented plant $\mathcal{P}^a$ is obtained:

$$\begin{bmatrix} \tilde{e} \\ \tilde{y} \end{bmatrix} = \begin{bmatrix} \mathcal{P}^a_{11} & \mathcal{P}^a_{12} \\ \mathcal{P}^a_{21} & \mathcal{P}^a_{22} \end{bmatrix} \begin{bmatrix} \tilde{w} \\ \tilde{u} \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} P_{11} & 0 \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} P_{12} & 0 \\ 0 & I \end{bmatrix} \\ \begin{bmatrix} P_{21} & 0 \\ 0 & I \end{bmatrix} & \begin{bmatrix} P_{22} & 0 \\ 0 & 0 \end{bmatrix} \end{bmatrix} \begin{bmatrix} \begin{bmatrix} w \\ cmd \end{bmatrix} \\ \begin{bmatrix} u \\ r \end{bmatrix} \end{bmatrix} \quad (12)$$

⋈

Remark 1. The stability conditions given by equations (8-10) are general necessary and sufficient conditions valid for any controller and plant.

Theorem 3.2 (General LFT $(P, \tilde{J}, Q)$.) *Given a generalized plant* $P \in \mathcal{R}_P$ *(with* $P_{22} = G_u \in \mathcal{R}_P$*), and a general stabilizing four-parameter controller* $K_F(Q) \in \mathcal{R}_P$*. The controller can be formulated as a lower linear fractional transformation,* $K_F(Q) = F_l(\tilde{J}, Q)$ *with* $Q \in \mathcal{RH}_\infty$*, without affecting the stability of the system. Furthermore, an alternative necessary and sufficient condition is for the transfer function matrix* T_P *of Figure 2 to belong to* $\mathcal{RH}_\infty$*.*

Proof: Using the augmented plant from equation (12) and a general coefficient matrix $\tilde{J} \in \mathcal{R}_P$ with appropriate dimensions, the general four-parameter controller $K_F(Q)$ of equation (6) can be formulated as in Figure 2 with $\tilde{u} = [u^\top \ r^\top]^\top$ and $\tilde{y} = [y^\top \ cmd^\top]^\top$.

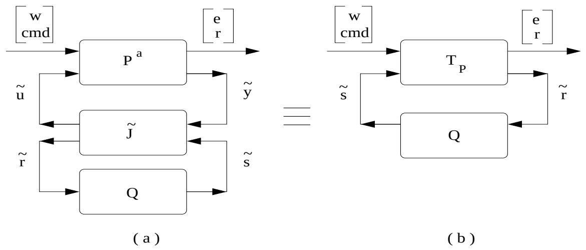

Figure 2: Stability $(\mathcal{P}^a, \tilde{J}, Q) \equiv$ Stability (T_P, Q).

Closing loops and after some manipulation, it is easy to obtain definitions for $\tilde{u}$ and $\tilde{y}$ based on $[\tilde{w}^\top \ \tilde{s}^\top]^\top$, where $\tilde{w} = [w^\top \ cmd^\top]^\top$:

$$\begin{aligned}\tilde{u} &= \tilde{J}_{11}\mathcal{P}^a_{21}\ \tilde{w} + \tilde{J}_{11}\mathcal{P}^a_{22}\ \tilde{u} + \tilde{J}_{12}\ \tilde{s} \\ &= \Delta_a^{-1}\ [\ \tilde{J}_{11}\mathcal{P}^a_{21}\ \tilde{w} + \tilde{J}_{12}\ \tilde{s}\ ]\end{aligned} \tag{13}$$

$$\begin{aligned}\tilde{y} &= \mathcal{P}^a_{21}\ \tilde{w} + \mathcal{P}^a_{22}\ \tilde{u} = \mathcal{P}^a_{22}\ \Delta_a^{-1}\ \tilde{J}_{12}\ \tilde{s} + \\ &\quad [\ \mathcal{P}^a_{21} + \mathcal{P}^a_{22}\ \Delta_a^{-1}\ \tilde{J}_{11}\mathcal{P}^a_{21}\ ]\ \tilde{w} +\end{aligned} \tag{14}$$

$$\Delta_a = (I - \tilde{J}_{11}\mathcal{P}^a_{22}) \tag{15}$$

An LFT constraint becomes apparent now, it is required that the inverse of Δ_a exists. The coefficient terms for the matrix T_P in Figure 2(b) are obtained using equations (12) and (13-14):

$$\begin{bmatrix}\tilde{e} \\ \tilde{r}\end{bmatrix} = \begin{bmatrix}\mathcal{P}^a_{11} + \mathcal{P}^a_{12}\ \Delta_a^{-1}\tilde{J}_{11}\mathcal{P}^a_{21} & \mathcal{P}^a_{12}\ \Delta_a^{-1}\tilde{J}_{12} \\ \tilde{J}_{21}\left(\ I + \mathcal{P}^a_{22}\ \Delta_a^{-1}\tilde{J}_{11}\ \right)\ \mathcal{P}^a_{21} & \tilde{J}_{22} + \tilde{J}_{21}\mathcal{P}^a_{22}\ \Delta_a^{-1}\tilde{J}_{12}\end{bmatrix}\begin{bmatrix}\tilde{w} \\ \tilde{s}\end{bmatrix} \tag{16}$$

The closed-loop transfer function $TF_{\tilde{w}\rightarrow\tilde{e}}$ for any $Q \in \mathcal{R}_P$ can be found by solving the system of equations given by equation (16) and $\tilde{s} = Q\ \tilde{r}$:

$$TF_{\tilde{w}\rightarrow\tilde{e}} = T_{P11} + T_{P12}Q(I - T_{P22}Q)^{-1}T_{P21} \tag{17}$$

The closed-loop will be stable if and only if $T_P \in \mathcal{RH}_\infty$ and $Q(I - T_{P22}Q)^{-1} \in \mathcal{RH}_\infty$. $\bowtie$

Remark 1. The equation for T_P given in equation (16), and that for the closed-loop $TF_{\tilde{w}\rightarrow\tilde{e}}$ from equation (17), are also general equations valid for any controller and general plant. Stability is not ensured unless an appropriate coefficient matrix $\tilde{J} \in \mathcal{R}_P$ and a suitable generalized plant $\mathcal{P}^a \in \mathcal{R}_P$ are chosen.

In the case of the four-parameter controller and Q, equations (1) and (2) the corresponding coefficient matrix $\tilde{J} = J_{4param}$ is given by:

$$\begin{bmatrix}u \\ r \\ \tilde{r} \\ w\end{bmatrix} = \begin{bmatrix}\tilde{V}^{-1}\tilde{U} & \tilde{V}^{-1}\tilde{U}_{ff} & \tilde{V}^{-1} & 0 \\ 0 & 0 & 0 & I \\ V^{-1} & -\tilde{N}_u\tilde{V}^{-1}\tilde{U}_{ff} & -\tilde{N}_u\tilde{V}^{-1} & 0 \\ 0 & I & 0 & 0\end{bmatrix}\begin{bmatrix}y \\ w \\ s \\ r\end{bmatrix} \tag{18}$$

The controller terms of equation (1) are recovered after some involved algebraic manipulations using several Bezout relations (i.e $N_u\tilde{V} - V\tilde{N}_u = 0 \rightarrow \tilde{N}_u\tilde{V}^{-1} = V^{-1}N_u$, $\tilde{M}V - \tilde{N}_uU = I \rightarrow \tilde{M} - \tilde{N}_uUV^{-1} = V^{-1}$ and $V\tilde{M} - N_u\tilde{U} = I \rightarrow N_u^{-1} = N_u^{-1}V\tilde{M} - \tilde{U}$).

Remark 2. The term T_{P22} in equation (16) is always zero regardless of the controller or the generalized plant used when no uncertainty is considered. This result can be easily obtained using $\hat{J} = J_{nom}$ (i.e. the Youla parameterization coefficient matrix) and $\mathcal{P}^a_{22} = G_u$:

$$\begin{aligned}\tilde{J}_{22} + \tilde{J}_{21}\mathcal{P}^a_{22}\ \Delta_a^{-1}\tilde{J}_{12} &= -V^{-1}N_u + \\ V^{-1}G_u\left(I - UV^{-1}G_u\right)^{-1}\tilde{V}^{-1} &= 0\end{aligned} \tag{19}$$

This implies that the nominal closed-loop transfer function is given by the familiar relationship:

$$TF_{\tilde{w}\rightarrow\tilde{e}} = \ \mathcal{F}_l(T_P, Q) = \ T_{P11} + T_{P12}\ Q\ T_{P21} \tag{20}$$

Remark 3. The uncertain case is equally obtained by using again $\tilde{J} = J_{nom}$ and substituting $\mathcal{P}^a_{22}$ by $G(S) = G_S = N_u(S)M(S)^1 = (N_u + VS)(M + US)^{-1}$ from reference [8], where S indicates the model uncertainty. In this case, the term T_{P22} is always equal to the uncertainty:

$$\begin{aligned}&\tilde{J}_{22} + \tilde{J}_{21}\mathcal{P}^a_{22}\ \Delta_a^{-1}\tilde{J}_{12} = -V^{-1}N_u + \\ &V^{-1}G_S\left(I - UV^{-1}G_S\right)^{-1}\tilde{V}^{-1} = -V^{-1}N_u + \\ &V^{-1}N_u(S)M(S)^{-1}\left(I - UV^{-1}G_S\right)^{-1}\tilde{V}^{-1} = \\ &-V^{-1}N_u + V^{-1}(N_u + VS) = S\end{aligned} \tag{21}$$

4 Integrated Control/Filter Architectures

In [8] an architecture for high-performance controllers was given to iteratively improve the closed-loop performance through the re-design of the Youla parameter Q_c, see Figure 3. This architecture is based on a modification of the Youla parameterization which allows for a separation principle on the controller. The separation principle that results from this architecture has the potential to address the trade-off between controller robustness and controller performance. From its connections with observer-based theory and the (generalized) internal model control principle, GIMC, this architecture presents as well with a natural way to implement fault diagnostic schemes.

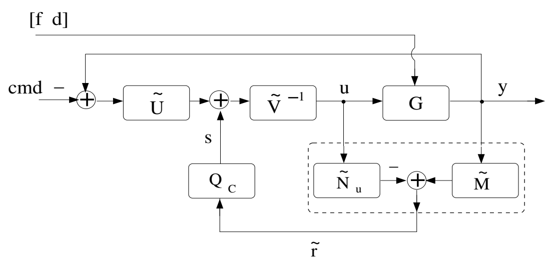

Figure 3: Fault Tolerant GIMC-Youla Structure, FT-GIMC.

As in the previous section an equivalent LFT can be obtained for this architecture based on a coefficient matrix

$J_{FT-GIMC} \in \mathcal{R}_P$ (a particular case of J_{4param}) and the Youla free-parameter $Q_c \in \mathcal{RH}_\infty$, reference [8]:

$$J_{FT-GIMC} = \begin{bmatrix} \tilde{V}^{-1}\tilde{U} & -\tilde{V}^{-1}\tilde{U} & \tilde{V}^{-1} \\ V^{-1} & \tilde{N}_u\tilde{V}^{-1}\tilde{U} & -V^{-1}N_u \end{bmatrix} \tag{22}$$

The FT-GIMC structure can be converted into a Residual Generation (RG-GIMC) paradigm by incorporating the residual generation parameter Q_f, see Figure 4.

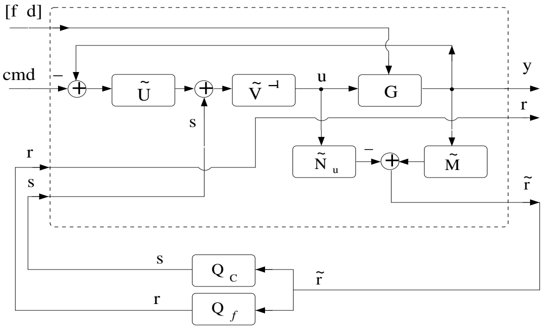

Figure 4: Standard RG-GIMC paradigm.

4.1 LFT formulation of the RG-GIMC Architecture

In order to develop the results presented in this section it is more useful to draw the architecture of Figure 4 as the familiar two-block diagram formed by the generalized plant P^a_{RG} and the integrated controller $K_{RG}(Q) \in \mathcal{R}_P$, see Figure 5. To achieve this, the exogenous inputs to the plant and controller, i.e. $[f^\top \; d^\top]^\top$ and cmd, are combined into a common exogenous plant input $\tilde{w}$. The controlled input to the plant becomes $\tilde{u} = [u^\top \; r^\top]^\top$ while the input to the controller is now given by $\tilde{y} = [y^\top \; cmd^\top]^\top$. This augmented plant is a particularization of $\mathcal{P}^a$, equation (12), where it is assumed $P_{11} = P_{12} = 0$, $P_{21} = [G_f \; G_d]$ and $P_{22} = G_u$.

$$\begin{bmatrix} r \\ \begin{bmatrix} y \\ cmd \end{bmatrix} \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & I \end{bmatrix} \\ \begin{bmatrix} G_f & G_d & 0 \\ 0 & 0 & I \end{bmatrix} & \begin{bmatrix} G_u & 0 \\ 0 & 0 \end{bmatrix} \end{bmatrix} \begin{bmatrix} \begin{bmatrix} f \\ d \\ cmd \end{bmatrix} \\ \begin{bmatrix} u \\ r \end{bmatrix} \end{bmatrix} \tag{23}$$

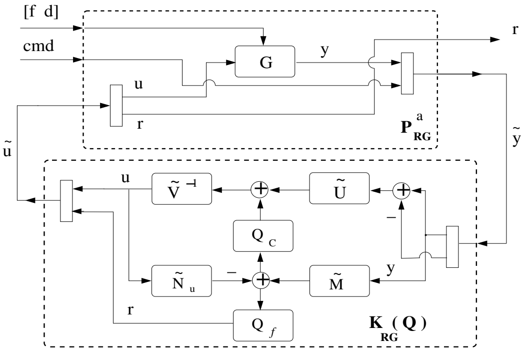

Figure 5: Closed-Loop $(K_{RG}(Q), P^a_{RG})$.

It is noted that sensor and actuator models are considered embedded in the plant G and that no general performance index has been defined for this generalized plant P^a_{RG}. Also, it is assumed that $P^a_{RG} \in \mathcal{R}_P$ is stabilizable and detectable and that all the instabilities, if any, are contained in $G_u \in \mathcal{R}_P$. The corresponding parameterization for the class of RG-GIMC controllers for a given plant is given by:

$$\begin{bmatrix} u \\ r \end{bmatrix} = \begin{bmatrix} (\tilde{V} + Q_c\tilde{N}_u)^{-1}(\tilde{U} + Q_c\tilde{M}) & -(\tilde{V} + Q_c\tilde{N}_u)^{-1}\tilde{U} \\ Q_f\left(\tilde{M} - \tilde{N}_u(\tilde{V} + Q_c\tilde{N}_u)^{-1}(\tilde{U} + Q_c\tilde{M})\right) & Q_f\tilde{N}_u(\tilde{V} + Q_c\tilde{N}_u)^{-1}\tilde{U} \end{bmatrix} \begin{bmatrix} y \\ cmd \end{bmatrix} \tag{24}$$

where $(Q_c, Q_f) \in \mathcal{RH}_\infty$ and $(\tilde{V} + Q_c\tilde{N}_u)(\infty)$ is invertible.

The associated controller lower LFT $K_{RG}(Q) = F_l(J_{RG-GIMC}, Q)$ is given by

$$\begin{bmatrix} u \\ r \\ \tilde{r} \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} \tilde{V}^{-1}\tilde{U} & -\tilde{V}^{-1}\tilde{U} & \begin{bmatrix} \tilde{V}^{-1} & 0 \end{bmatrix} \\ 0 & 0 & \begin{bmatrix} 0 & I \end{bmatrix} \end{bmatrix} \\ V^{-1} \quad \tilde{N}_u\tilde{V}^{-1}\tilde{U} \quad -\begin{bmatrix} V^{-1}N_u & 0 \end{bmatrix} \end{bmatrix} \tag{25}$$

$$\tilde{s} = \begin{bmatrix} s \\ r \end{bmatrix} = Q\,\tilde{r} = \begin{bmatrix} Q_c \\ Q_f \end{bmatrix} \tilde{r} \tag{26}$$

Note that $J_{RG-GIMC} \in \mathcal{R}_P$, equation (25), is exactly as the coefficient matrix $J_{FT-GIMC}$, equation (22), augmented by the residual channel r and is a particular case of J_{4param}, equation (18), without the exogenous Q-parameters, i.e. Q_{cw}, Q_{fw}.

Assume the RG-GIMC plant P^a_{RG} from equation (23) is augmented with a general performance index, $e = P_{11}\,w + P_{12}\,u$, and $P_{21} = [G_f \; G_d]$ and $P_{22} = G_u$ as before. The corresponding coefficient matrix $T_{RG-GIMC}$ is given by:

$$\begin{bmatrix} \begin{bmatrix} e \\ r \end{bmatrix} \\ \tilde{r} \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} P_{11} + P_{12}M\tilde{U}P_{21} & -P_{12}M\tilde{U} \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} P_{12}M & 0 \\ 0 & I \end{bmatrix} \\ \begin{bmatrix} \tilde{M}P_{21} & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 \end{bmatrix} \end{bmatrix} \begin{bmatrix} \begin{bmatrix} w \\ cmd \end{bmatrix} \\ \begin{bmatrix} s \\ r \end{bmatrix} \end{bmatrix} \tag{27}$$

And the nominal RG-GIMC closed-loop $TF_{\tilde{w}\to\tilde{e}}$ by:

$$\begin{bmatrix} e \\ r \end{bmatrix} = \begin{bmatrix} P_{11} + P_{12}M(\tilde{U} + Q_c\tilde{M})P_{21} & -P_{12}M\tilde{U} \\ Q_f\tilde{M}P_{21} & 0 \end{bmatrix} \begin{bmatrix} w \\ cmd \end{bmatrix} \tag{28}$$

Remark 1. As expected, the closed-loop matrix obtained is affine in $Q = \left[\begin{smallmatrix} Q_c \\ Q_f \end{smallmatrix}\right]$. This is observed better by rewriting equation (28) as follows:

$$TF_{\tilde{w}\to\tilde{e}} = \begin{bmatrix} P_{11} + P_{12}M\tilde{U} & -P_{12}M\tilde{U} \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} P_{12}M & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} Q_c \\ Q_f \end{bmatrix} \begin{bmatrix} \tilde{M}P_{21} & 0 \end{bmatrix} \tag{29}$$

As it was noted in references [4, 5], for the nominal case each of the closed-loop transfer functions in equations (28,29) is only affected by one and only one of the free parameters. The feedback controller contribution is affine in Q_c while the diagnostic contribution of the controller is linear in Q_f. Finally, note that the $TF_{w\to e}$ coefficient in equation (28) is equal to the closed-loop transfer function corresponding to the Youla parameterization after using the double Bezout equation $U\tilde{M} - M\tilde{U} = 0$.

The duple $(T_{RG-GIMC}, Q)$ provides the basic architecture for an integrated control and diagnosis filter design while enabling the definition of general performance indexes by appropriate choice of $e = [e_c^\top \quad e_f^\top]^\top$. For example, the filter performance channel can be defined in a model-matching format $e_f = \mathcal{V}_f f - r$ where the transfer function $\mathcal{V}$ is used to define the different FDI problems and to shape the ideal fault model [3]. Additionally, a corresponding model-matching index can be defined for the controller performance channel, $e_c = y - \mathcal{V}_c\, cmd$. The RG-GIMC plant is defined by equation (12) with $P_{11} = P_{21} = [G_f \; G_d]$ and $P_{12} = P_{22} = G_u$.

Note that if an optimization-based approach is applied to solve the integrated controller design problem, both performance indexes will be equally minimized. This is not usually desirable, since the controller performance index might be required to be always below a minimum bound to ensure for example stability of the system while the residual performance might be allowed to degrade. There are several ways to address this trade-off, the most general involves using weights to penalize more those channels that are critical for the control performance. Another approach, proposed in [11], involves a two-step sequential optimization where first an initial bound γ_1 is obtained for the control part alone, and second the integrated controller/filter performance is minimized using the previous control bound so that the residual performance channel is penalized.

Using the previous results the coefficient matrix T_{sp_2} for the latter approach combined with the fault and control model-matching cases is given below:

$$\begin{bmatrix}\begin{bmatrix}e_c\\ e_f\end{bmatrix}\\ \tilde{r}\end{bmatrix} = \begin{bmatrix}\begin{bmatrix}V\tilde{N}_f & V\tilde{N}_d & -(N_u\tilde{U} + \mathcal{V}_c)\\ \mathcal{V}_f & 0 & 0\end{bmatrix} & \begin{bmatrix}N_u & 0\\ 0 & -\gamma_1\end{bmatrix}\\ \begin{bmatrix}\tilde{N}_f & \tilde{N}_d & 0\end{bmatrix} & \begin{bmatrix}0 & 0\end{bmatrix}\end{bmatrix} \begin{bmatrix}\begin{bmatrix}f\\ d\\ cmd\end{bmatrix}\\ \begin{bmatrix}s\\ r\end{bmatrix}\end{bmatrix} \tag{30}$$

The effects of the different inputs on the different outputs and performance channels become transparent in the above equation. Note again that the $(2,2)$ term is zero since there is no uncertainty. The closed-loop transfer matrix $TF_{\tilde{w}\to\tilde{e}}$ is easily obtained (the resulting equations have been re-arranged to emphasize the decoupling nature between the free parameters in the nominal case):

$$TF_{\tilde{w}\to\tilde{e}} = \begin{bmatrix}V\tilde{N}_f & V\tilde{N}_d & -(N_u\tilde{U} + \mathcal{V}_c)\\ \mathcal{V}_f & 0 & 0\end{bmatrix} + \begin{bmatrix}N_u & 0\\ 0 & -\gamma_1\end{bmatrix}\begin{bmatrix}Q_c & 0\\ 0 & Q_f\end{bmatrix}\begin{bmatrix}\tilde{N}_f & \tilde{N}_d & 0\\ \tilde{N}_f & \tilde{N}_d & 0\end{bmatrix} \tag{31}$$

The separation principle for the nominal closed-loop design studied in [9, 5] becomes clear. As mentioned before, the closed-loop is given in a form affine to the diagonal augmentation of the Youla control and diagnostic parameters (Q_c, Q_F) which is conducive to the application of numerical optimization techniques [1]. Nevertheless, it is highlighted that for the nominal integrated controller, the diagnostic signal is coupled to both free Youla parameters, see equation (4).

5 Conclusions

In this paper general stability conditions and formulae for the linear fractional transformation formulation of the well-known four-parameter controller have been given. The LFT formulation allows for a more transparent look at the interactions between the different objectives. It was also shown how several architectures for fault tolerant control and fault detection & identification can also be derived within the LFT formulation.

This research was supported under NASA Langley Cooperative Agreement No. NCC-1-337, technical contract monitor Dr. Christine Belcastro.

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