High-Performance Architecture for Design and Analysis of Robust Anti-Windup Compensators

Andrés Marcos, Matthew C. Turner and Ian Postlethwaite

***Abstract*— In this paper a general framework for the design and analysis of robust anti-windup compensators is presented. The proposed framework combines the Weston-Postlethwaite anti-windup scheme with ideas from residual generation and robust high-performance control architectures. It is shown that the framework is well connected to the Youla controller parameterization and to fault tolerant/detection schemes. Furthermore, the proposed framework provides a transparent analysis of the interactions between the different design parameters which allows for a clear design trade-off between robust stability and robust performance for the saturated and unsaturated closed-loops.**

I. INTRODUCTION

Anti-windup (AW) compensation is a common approach used by control engineers to cope with actuator saturation and many methods are available to assist with its construction (see, for example [2], [6], [11] and references therein). With few exceptions, most available methods tackle the problems of stability and performance by (implicitly) assuming that the AW design inherits the robustness properties of the robust linear system [7], [13]. This makes some intuitive sense and indeed, if the uncertainty present in the real system is sufficiently small, standard AW techniques can be applied with some confidence. In [10] it was argued that robust stability of the linear system was only a *necessary* condition for robust stability of the saturated closed-loop system. Indeed, in that reference it was demonstrated that a saturated closed-loop system which behaved well using a “good” static AW design was actually destabilised when uncertainty was introduced.

Recently, several researchers [5], [10], [4] have approached the robust anti-windup problem by trying to incorporate robustness directly into the design of the anti-windup compensator or by adding additional filters to improve the robustness of the nominal AW compensator.

The aim of this paper is to present a general framework for robust anti-windup design and analysis. The architecture proposed combines the Weston-Postlethwaite anti-windup (W&P-AW) scheme with ideas from residual generation and robust high-performance control. It is shown that the robust anti-windup design is dependent on three design parameters: two arising from the left and right coprime factors of the nominal plant and the third being the well-known free-parameter from the Youla controller parameterization.

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II. ANTI-WINDUP PROBLEM DEFINITION

Traditionally, the aim of AW compensation was rather subjective, although now [7], [13], [14] it is accepted to mean the modification of a nominal (linear) controller – with good performance and robustness in the unconstrained case– so that:

1. The desired closed-loop performance and robustness are achieved when the initial conditions and reference signals do not cause saturation.
2. In the event of saturation, the constrained response is close to the unconstrained behaviour (although obviously limited by infeasible demands).
3. The recovery of the desired characteristics after saturation is fast.

A scheme that represents most anti-windup configurations is given in [13], see Figure 1, where an interpretation of the conditioning for any controller $K(s) = [K_w(s)\ K_o(s)]$ is given in terms of a single transfer function $M(s)$ (which is actually a right coprime factor of the nominal input plant $G_o(s)$ where $G(s) = [G_d(s)\ G_o(s)] \in \mathcal{RH}_\infty$). For global stability guarantees, the generalized plant $G(s)$ is assumed exponentially stable.

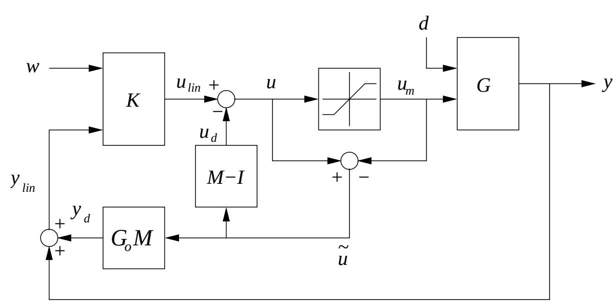

Fig. 1. Weston-Postlethwaite AW scheme.

The signal $w \in \mathbb{R}^{n_w}$ is the reference command, $d \in \mathbb{R}^{n_d}$ is the disturbance on the plant and $y \in \mathbb{R}^p$ is the plant output. The control signal is $u \in \mathbb{R}^m$ and the actual plant input is $\text{sat}(u) = u_m \in \mathbb{R}^m$. The signals $u_d \in \mathbb{R}^m$ and $y_d \in \mathbb{R}^p$ are produced by the anti-windup compensator $[(M-I)'\ (G_oM)']'$.

The saturation and deadzone functions can be defined as

$$\mathrm{sat}(u) = \begin{bmatrix} \mathrm{sat}_1(u_1) \\ \vdots \\ \mathrm{sat}_m(u_m) \end{bmatrix} \quad \mathrm{Dz}(u) = \begin{bmatrix} \mathrm{Dz}_1(u_1) \\ \vdots \\ \mathrm{Dz}_m(u_m) \end{bmatrix} \quad (1)$$

where for all $i \in \{1,\ldots,m\}$ the saturation nonlinearity is given by $\mathrm{sat}_i(u_i) = \mathrm{sign}(u_i)\min(|u_i|, \bar{u_i})$; the dead-zone by $\mathrm{Dz}_i(u_i) = \mathrm{sign}(u_i)\max(0, |u_i| - \bar{u_i})$; and $\bar{u_i} > 0$ is a fixed bound. Relating both functions through the identity $\mathrm{sat}_i(u_i) = u_i - \mathrm{Dz}_i(u_i)$, it can be shown that Figure 1 can be re-drawn as in Figure 2.

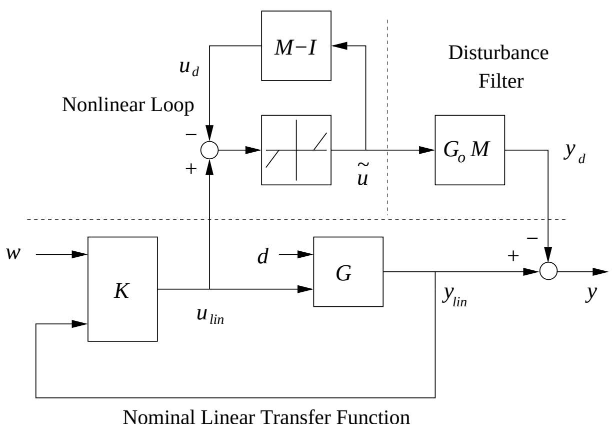

Fig. 2. W&P-AW analytical representation.

Figure 2 reveals an attractive decoupling into a nominal linear system, a nonlinear loop, and a disturbance filter. Note that if no saturation occurs ($\tilde{u} = 0$), then the nominal linear system is all that is required to determine the system's behaviour. However, if saturation occurs ($\tilde{u} \neq 0$), the nonlinear loop and disturbance filter become active. Using this representation, the question of global stability for the complete system is translated into determining whether the nonlinear loop is stable. Also, note that the dynamics of the disturbance filter determine the manner in which the nominal linear behaviour is affected during and after saturation.

It is well understood that the model of the system, $G(s)$ is not a perfect representation of the true system and that uncertainty will inevitably be present. Unfortunately, as stressed in [11], most anti-windup schemes assume that the model of the plant is a good representation of the true system and hence do not account for uncertainty explicitly.

In order to see the difficulty which arises from the presence of uncertainty, consider the uncertain plant given by $\tilde{G}(s) = [G_d(s) \;\; G_o(s) + \Delta_G(s)]$ (we are assuming additive uncertainty since it is one of the most general and widely used representations for uncertainty in engineering). Starting from Figure 1 with $G(s)$ replaced by $\tilde{G}(s)$ and following the loop manipulations from [10], it is easy to re-draw that configuration as in Figure 3.

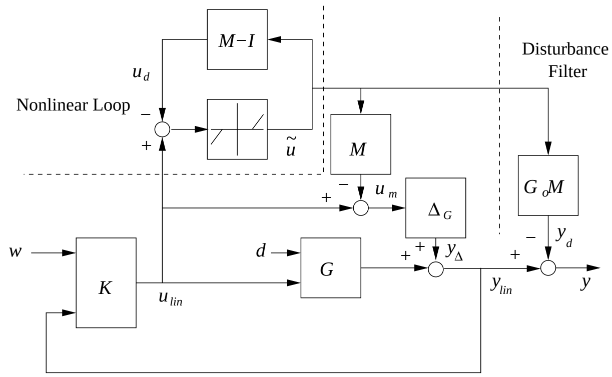

Fig. 3. Uncertain W&P-AW analytical set-up.

Note that the term $\Delta_G M : \tilde{u} \mapsto y_\Delta$ destroys the decoupling of the linear system and nonlinear loop. This loss of decoupling forces several assumptions in order to obtain global stability results, mainly that the uncertainty $\Delta_G(s)$ is asymptotically stable and small in some sense. This in turn forces a Small-Gain argument for the stability of the saturated system which reduces its general applicability (to systems within the family obtained from the reduced uncertainty).

Based on the presence of uncertainty, a robust anti-windup problem [5], [10] can be defined as requiring fulfilment of the three initial "nominal" anti-windup objectives for all uncertain plants within the family $\tilde{G}(s)$ – where the uncertainty $\Delta_G(s) \in \mathcal{RH}_\infty$ is bounded $||\Delta_G(s)||_\infty \leq 1$. A so-called "weakened" version of the robust AW problem [4] is similarly formulated but requires only nominal performance and robust stability (hence, the weakened condition with respect to robust performance).

III. High-Performance Control Architecture

Before presenting the robust AW scheme, the high-performance control architecture [9], [16] from which the proposed scheme arises is reviewed. 'High-performance' in this context is defined as shifting the focus on the standard performance/robustness trade-off to the performance side (including disturbance rejection).

Let the nominal plant $G(s)$ and controller $K(s)$ have the following right and left coprime factorizations:

$$G(s) = [G_d \;\; G_o] = [N_d \;\; N_o]\, M^{-1} = \tilde{M}^{-1}[\tilde{N}_d \;\; \tilde{N}_o] \quad (2)$$

$$K(s) = [K_w \;\; K_o] = [U_w \;\; U_o]\, V^{-1} = \tilde{V}^{-1}[\tilde{U}_w \;\; \tilde{U}_o] \quad (3)$$

where $N_o, M, \tilde{N}_o, \tilde{M}, U_o, V, \tilde{U}_o, \tilde{V} \in \mathcal{RH}_\infty$ must satisfy the double coprime factorization

$$\begin{bmatrix} \tilde{V} & -\tilde{U}_o \\ -\tilde{N}_o & \tilde{M} \end{bmatrix} \begin{bmatrix} M & U_o \\ N_o & V \end{bmatrix} = \begin{bmatrix} M & U_o \\ N_o & V \end{bmatrix} \begin{bmatrix} \tilde{V} & -\tilde{U}_o \\ -\tilde{N}_o & \tilde{M} \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} \quad (4)$$

Consider the system $\tilde{G}(s) = G_o(s) + \Delta_G(s)$ (to simplify the presentation it is assumed, without loss of generality, that there are no disturbances). The Youla parameterization [15] describes the class of all proper controllers that stabilise the system in terms of a free-parameter $Q_c(s) \in \mathcal{RH}_\infty$:

$$K_o(Q_c) = (\tilde{V} + Q_c\tilde{N}_o)^{-1}(\tilde{U}_o + Q_c\tilde{M}) \tag{5}$$

such that $det(\tilde{V} + Q_c\tilde{N}_o)(\infty) \neq 0$.

Using an alternative implementation of the above Youla parameterization, reference [9] proposed an architecture for high-performance controllers, see Figure 4. This architecture allows for a separation principle on the controller and has connections with the generalised internal model control (GIMC) [16].

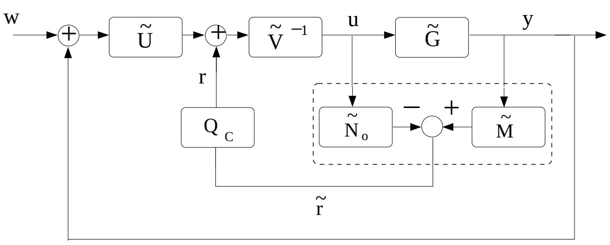

Fig. 4. High-performance GIMC-Youla Structure.

As remarked in [16], the architecture from Figure 4 is based on the concept of residual generators (dependent on a model of the system and thus related to the internal model control approach). This concept is well-known in the field of fault detection and isolation (FDI) where it is used to detect and isolate faults in a monitored system using its input and output signals. A general frequency residual generator can be obtained using the left coprime factorization of the nominal plant $G_o(s) = \tilde{M}^{-1}\tilde{N}_o$ [3], [1]:

$$\begin{aligned} r &= Q_c\,\tilde{r} = Q_c\,(\tilde{N}_o u - \tilde{M}y\,) \\ &= Q_c\,(\tilde{N}_o\,u - \tilde{N}_o\,u - \tilde{M}\Delta_G u) = -Q_c\tilde{M}\Delta_G u \end{aligned} \tag{6}$$

For FDI purposes, the filter $Q_c(s)$ is chosen to minimise the effects of uncertainty and disturbances while maximizing that of the faults (also present in $\tilde{r}(s)$ if there are any) but its output signal $r(s)$ is not fed back to the controller. On the other hand, within the high-performance framework, the Youla parameter $Q_c(s)$ is interpreted as a robust controller that only takes action in the feedback loop whenever exogenous signals (e.g. disturbances and faults) or model uncertainties are present, i.e when the primary residual $\tilde{r}(s)$ is non-zero.

Note that in the latter fashion, the design of the nominal controller $K_o(s)$ (hence, of the nominal performance characteristics for the closed loop) is independent of the design of the robustifying controller $Q_c(s)$ (which sets the closed-loop robust performance).

The relation of the architecture in Figure 4 with the Youla parameterization is clear, i.e. the transfer function from the exogenous signal $w(s)$ and plant output $y(s)$ to the control output signal $u(s)$ is:

$$\begin{aligned} u &= \tilde{V}^{-1}\Big(\tilde{U}_w\,w + \tilde{U}_o\,y + Q_c\tilde{M}\,y - Q_c\tilde{N}_o\,u\Big) \\ &= (\tilde{V} + Q_c\tilde{N}_o)^{-1}\Big(\tilde{U}_w\,w + (\tilde{U}_o + Q_c\tilde{M})\,y\Big) \end{aligned} \tag{7}$$

Notice that the second term of the above equation, $\mathcal{T}_{R.P.} : u \rightarrow y$, is indeed equal to the Youla controller parameterization from equation (5).

IV. A General Robust AW Framework

In this section the proposed framework for the design and analysis of robust AW compensators is given. The main idea is to combine the strengths of the W&P-AW scheme from Section II (its ease of analysis and the decoupling between the nominal linear and nonlinear loops) with the advantages of the previous high-performance architecture (its decoupled design for the nominal and robust performance objectives).

It is noted that the W&P-AW scheme is the dual of Kothare's unified conditioning approach [7], [13], and that all the linear anti-windup schemes can be represented in terms of the latter, thus the proposed framework is applicable to all AW linear schemes.

Using the plant input $u_m(s)$ and output $y(s)$ as the inputs to the residual generator from equation (6) and assuming the output $r(s)$ of the robustifying controller $Q(s)$ is directly fed to the plant output $y(s)$, the schemes from Figures 1 and 4 are easily combined to form the proposed robust AW architecture, Figure 5.

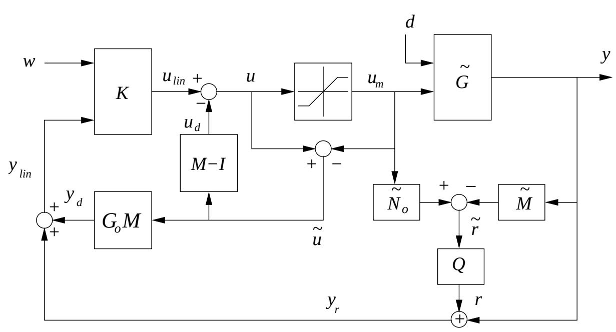

Fig. 5. General Robust AW architecture.

From direct observation of the configuration in Figure 5 it is clear how this robust AW scheme operates in order to achieve its objectives in the presence of saturation and/or uncertainty:

- Case 1. No uncertainty ($\tilde{r} = 0$), no saturation ($\tilde{u} = 0$). The nominal linear loop (G_o, K) retains the performance and stability objectives for which it was designed.
- Case 2. No uncertainty ($\tilde{r} = 0$), saturation ($\tilde{u} \neq 0$). The configuration from Figure 1 is obtained – with the aforementioned advantages for anti-windup synthesis and analysis.

Case 3. Uncertainty ($\tilde{r} \neq 0$), no saturation ($\tilde{u} = 0$). The filter $Q\tilde{M}$ becomes a robustifying controller which minimizes the uncertainty effect on the nominal loop (G_o, K), see Figure 6.

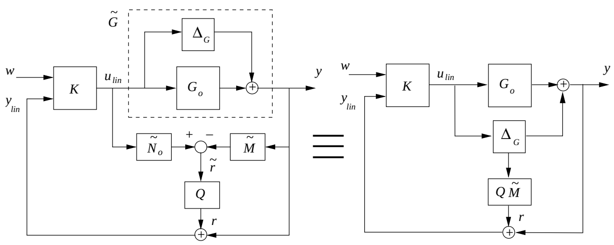

Fig. 6. Robustifying controller.

Case 4. Uncertainty ($\tilde{r} \neq 0$) and saturation ($\tilde{u} \neq 0$). Similarly to the previous case, the filter $Q\tilde{M}$ represents an additional degree-of-freedom to robustify the saturated closed-loop.

Since one of the goals of the proposed robust AW scheme is to incorporate the "nice" analytical nature of W&P-AW, the same manipulations required to obtain the input-output maps in Figure 2 are followed to obtain the analytical configuration of Figure 7.

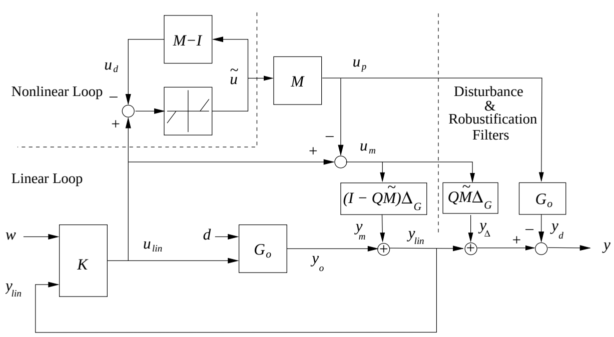

Fig. 7. Analytical representation of the robust AW architecture.

Not surprisingly, the analytical scheme in Figure 7 can be divided into similar partitions as Figure 2: linear loop, nonlinear loop, and disturbance (now also with robustification) filters.

It is observed that the linear and nonlinear loops are still coupled although this time through the term $y_m = (I - Q\tilde{M})\Delta_G u_m$. Also, there is a further coupling on the outer-loop $y = y_{lin} + y_\Delta - y_d$ due to the robustifying filter $y_\Delta = Q\tilde{M}\Delta_G u_m$. A large amount of design freedom lies in the free parameter, $Q(s)$, and there are two particularly striking choices:

1) $Q = 0$. In this case the results of [10] and thus Figure 2 are recovered; the coupling with the outer-loop is lost while the coupling between the linear and nonlinear loop occurs due to the uncertainty $\Delta_G(s)$. This implies that nominal stability is affected by the presence of uncertainty and saturation, and therefore the design of AW compensators need to account for this coupling. Furthermore, global stability conditions are limited by the Small-Gain Theorem – see Section II. This is the set-up used recently in a robust anti-windup LMI approach [10], which tries to optimize the stability robustness by minimizing the $\mathcal{L}_2$ gain of the map $\mathcal{T}_r : u_{lin} \mapsto u_m$.
2) $Q = \tilde{M}^{-1}$. In this case a complete decoupling of the linear and nonlinear loops is obtained. Indeed, this case represents a loop shifting of the uncertainty coupling from the inner linear loop to the outer-loop. This has obvious advantages as in this case a Small-Gain condition for global stability of the saturated system is *not necessary* (although it is desirable for robust performance). Moreover, this implies that minimization of the map $\mathcal{T}_p : u_{lin} \mapsto y_d$ (as in [11]) would automatically result in a stable system, regardless of the uncertainty present.

Therefore, choosing $Q = \tilde{M}^{-1}$ implies essentially unconditional robust stability at the expense of robust performance; while $Q = 0$, implies the potential of nominal robustness recovery during saturation and nominal linear behaviour in the absence of saturation.

In terms of analysing the performance of the anti-windup design (choice of $M(s)$), the three operational modes from [13] can be followed for the nominal and uncertain systems. These modes correspond to the AW objectives from Section II and measure the AW performance based on the time spent in modes *II* and *III* (i.e. time spent during saturation t_{sat} and time spent after saturation to recover linear behaviour t_{rec}, respectively). The difference in both times between the nominal ($\Delta_G(s) = 0$) and uncertain cases ($\Delta_G(s) \neq 0$) indicate the degradation on the AW performance due to the presence of uncertainty (ideally, the total difference should be minimum).

V. Synthesis of the Design Parameters

Roughly speaking, from Figure 7, it is noted that $M(s)$ can be designed to tackle standard AW performance and stability goals, i.e. minimizing the map $\mathcal{T}_p : u_{lin} \rightarrow y_d$; and the choice of $Q(s)$ can be used to trade-off robust stability and robust performance (of the unsaturated and saturated closed-loop) in a clear manner, i.e. minimising $\mathcal{T}_{RS} : u_{lin} \rightarrow y_m$ versus minimizing $\mathcal{T}_{RP} : u_{lin} \rightarrow y_\Delta$. Furthermore, from Figure 5, the factors $\tilde{N}_o(s)$ and $\tilde{M}(s)$ can be selected so that the primary residual signal $\tilde{r}(s)$ better captures the uncertainty (or exogenous disturbances and faults).

Indeed, in [13] it was noted that a possible choice for $M(s)$ was one of the right coprime factor for the nominal plant $G_o(s)$. Similarly, in [1], [16] it was shown that the parameters $\tilde{M}(s)$ and $\tilde{N}_o(s)$ where chosen to be the left coprime factors of the nominal plant. Therefore, before providing guidelines for the synthesis of the above design parameters ($M(s)$, $\tilde{M}(s)$, $\tilde{N}_o(s)$,

$Q(s)$), the connection of the robust AW scheme with the Youla parameterization is highlighted again –by deriving the equivalent transfer function to the mapping from equation (7).

A. Relationship to Youla parametrization

Starting from Figure 5, using the coprime factorizations of the nominal plant $G_o(s)$ and controller $K_o(s)$ from equations (2-3), and including the AW signal $\tilde{u}(s)$ in the transfer function, yields:

$$u = u_{lin} - u_d = u_{lin} - (M - I\,)\,\tilde{u} \tag{8}$$

$$\begin{aligned} u_{lin} =&\tilde{V}^{-1}\Big(\tilde{U}_w w + \tilde{U}_o y_d + \tilde{U}_o y + \tilde{U}_o r\,\Big) \\ =&\tilde{V}^{-1}\Big(\tilde{U}_w w + \tilde{U}_o G_o M\,\tilde{u} + \tilde{U}_o y \\ &+ \tilde{U}_o Q \tilde{N}_o u_m - \tilde{U}_o Q \tilde{M} y\,\Big) \end{aligned} \tag{9}$$

Using the relationship between saturated and unsaturated signals from equation (8) together with the signal $u_d(s) = (M(s) - I\,)\tilde{u}(s)$ gives:

$$\tilde{u} = u - u_m = (u_{lin} - u_d) - u_m \Rightarrow u_m = u_{lin} - M\tilde{u} \tag{10}$$

which substituted in equation (9) yields:

$$\begin{aligned} \tilde{V} u_{lin} =&\tilde{U}_w w + (\tilde{U}_o - \tilde{U}_o Q\tilde{M})y + \tilde{U}_o Q \tilde{N}_o u_{lin} \\ &+ (\tilde{U}_o G_o M - \tilde{U}_o Q \tilde{N}_o M)\tilde{u} \end{aligned} \tag{11}$$

Finally, noting $(\tilde{U}_o G_o M - \tilde{U}_o Q \tilde{N}_o M) = (\tilde{U}_o - \tilde{U}_o Q \tilde{M}) G_o M$, we get:

$$\begin{aligned} u_{lin} =& (\tilde{V} - \tilde{U}_o Q \tilde{N}_o)^{-1}\Big(\tilde{U}_w w + (\tilde{U}_o - \tilde{U}_o Q\tilde{M})y\,\Big) \\ &+ (\tilde{V} - \tilde{U}_o Q \tilde{N}_o)^{-1}(\tilde{U}_o - \tilde{U}_o Q\tilde{M}) G_o M \tilde{u} \end{aligned} \tag{12}$$

Now, in order to compare properly with the high-performance architecture from Section III, substitute $Q_c = -\tilde{U}_o^{-1} Q$ in the previous equation:

$$\begin{aligned} u_{lin} =& (\tilde{V} + Q_c \tilde{N}_o)^{-1}\Big(\tilde{U}_w w + (\tilde{U}_o + Q_c\tilde{M})y\,\Big) \\ &+ (\tilde{V} + Q_c \tilde{N}_o)^{-1}(\tilde{U}_o + Q_c\tilde{M}) G_o M\,\tilde{u} \end{aligned} \tag{13}$$

which compared to equation (7), represents the simplified two degrees-of-freedom Youla parameterization with an additional term. This additional term is indeed equal to the Youla parameterization in equation (5) but driven by the anti-windup signal $y_d(s) = G_o M\tilde{u}(s)$.

This connection to the Youla parameterization reinforces the design guidelines found in [13] for the anti-windup parameter $M(s)$, and in [1], [16] for the residual generator parameters $\tilde{N}_o(s)$ and $\tilde{M}(s)$. That is, they are chosen as the standard coprime factors:

$$\begin{bmatrix} M - I \\ N_o \end{bmatrix} = \left[\begin{array}{c|c} A + BF & B \\ \hline F & 0 \\ C + DF & D \end{array}\right] \tag{14}$$

$$\begin{bmatrix} \tilde{N}_o & \tilde{M} \end{bmatrix} = \left[\begin{array}{c|cc} A + LC & B + LD & L \\ \hline C & D & I \end{array}\right] \tag{15}$$

where $F(s)$ and $L(s)$ are the state feedback and observer gains chosen so that $(A + BF)$ and $(A + LC)$ are stable and also satisfy requirements on damping and frequency for nominal performance. The state-space matrices (A, B, C, D) come from a minimal realisation of $G_o(s)$.

B. Design guidelines

Using the previous guidelines and comments, the following algorithm is proposed for the design of robust AW compensators:

Step 1. Design a 2 Degrees-of-Freedom (DoF) controller $K(s) = [K_w \;\; K_o]$ to satisfy the required performance and stability objectives for the nominal plant $G_o(s)$ under no saturation.

Step 2. Design a nominal AW compensator (i.e. design $M(s) \equiv$ design F in equation (14)) to satisfy the nominal anti-windup objectives: i) minimization of the deviation between the behaviour of the nonlinear and linear loops during saturation; ii) fast recovery of the linear behaviour after saturation.

Step 3. Design $Q(s)$ and coprime factors $\tilde{M}(s), \tilde{N}_o$(s) (i.e. find $Q(s)$ and L in equation (15)) so that the nominal performance and stability objectives for the unsaturated and saturated cases are minimally degraded in the presence of uncertainty.

From Figure 7, and as mentioned at the beginning of the section, the two key robust stability and robust performance mappings are $\mathcal{T}_{RS} : u_{lin} \to y_m$ and $\mathcal{T}_{RP} : u_{lin} \to y_\Delta$. Thus essentially Step 3 of the design guidelines reduces to finding

$$\mu := \min_{Q(s),\tilde{M}(s)\in\mathcal{RH}_\infty} \left\| \begin{bmatrix} W_1 \mathcal{T}_{RS} \\ W_2 \mathcal{T}_{RP} \end{bmatrix} \right\|_{i,2} \tag{16}$$

where W_1 and W_2 are the linear operators associated with some appropriate transfer function matrices[1].

This is a difficult optimization problem; however note that

$$\begin{aligned} \mu =& \min_{Q(s),\tilde{M}(s)\in\mathcal{RH}_\infty} \left\| \begin{bmatrix} W_1(I - Q\tilde{M}) \\ W_2 Q\tilde{M} \end{bmatrix} \Delta_G \mathcal{T}_r \right\|_{i,2} \\ \leq& \min_{Q(s),\tilde{M}(s)\in\mathcal{RH}_\infty} \left(\left\| \begin{bmatrix} W_1(I - Q\tilde{M}) \\ W_2 Q\tilde{M} \end{bmatrix} \right\|_\infty \|\Delta_G\|_\infty \|\mathcal{T}_r\|_{i,2} \right) \\ =& \|\Delta_G\|_\infty \|\mathcal{T}_r\|_{i,2} \underbrace{\min_{Q(s),\tilde{M}(s)\in\mathcal{RH}_\infty} \left\| \begin{bmatrix} W_1(I - Q\tilde{M}) \\ W_2 Q\tilde{M} \end{bmatrix} \right\|_\infty}_{\gamma_{RS/RP}} \end{aligned} \tag{17}$$

So Step 3 of the guidelines reduces to the standard $\mathcal{H}_\infty$ optimization problem:

$$\gamma_{RS/RP} = \min_{Q(s),\tilde{M}(s)\in\mathcal{RH}_\infty} \left\| \begin{bmatrix} W_1(I - Q\tilde{M}) \\ W_2 Q\tilde{M} \end{bmatrix} \right\|_\infty \tag{18}$$

[1]We do not distinguish between a rational linear operator and its transfer function.

As is standard in the AW literature, the plant $\tilde{G}(s)$ is assumed to be exponentially stable for global stability guarantees[2]. Thus, the implementation of the residual generator could be given by $\tilde{r}(s) = Q(s)(G_o(s)u_m(s) - y(s))$, i.e. $\tilde{N}_o(s) = G_o(s)$ and $\tilde{M}(s) = I$. In this case, the third step of the above algorithm focuses on $Q(s)$ and our optimization problem reduces to that of seeking

$$Q^\star(s) = \arg \min_{Q(s)\in\mathcal{RH}_\infty} \left\| \begin{bmatrix} W_1(I-Q) \\ W_2 Q \end{bmatrix} \right\|_\infty \tag{19}$$

which should typically yield low-order $Q(s)$ as the weights $W_1(s), W_2(s)$ would typically also be low order.

Furthermore, using this special factorization for the residual generator we can relate our approach to the so-called weakened AW approach [4], developed using the filter $F(s) \equiv I - Q(s)$. Results on this relationship will be reported elsewhere.

An example of the proposed approach applied to robustification of static and low-order AW designs is presented in [8].

VI. Practical Implementation of the Robust AW Architecture

As a final remark, note that the direct implementation of the five-block robust AW compensator shown in Figure 5 will increase the total number of states in the closed loop by n_q states from $Q(s)$ plus five times the nominal plant states n_p since there are independent blocks for $(M(s) - I)$, $\tilde{N}_o(s)$, $\tilde{M}(s)$ and $G_o(s)M(s)$. This can be largely reduced following the standard implementation of the coprime factors as given in the formulae above, see Figure 8.

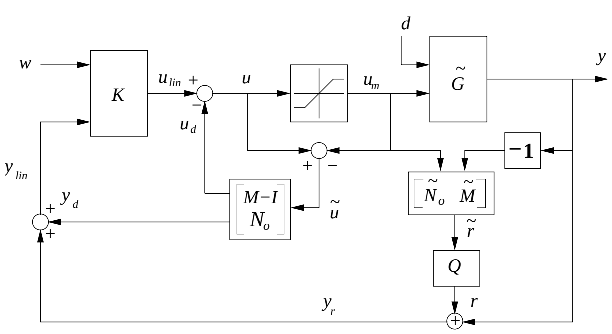

Fig. 8. Practical implementation of the robust AW architecture.

Further state reductions can be accomplished by using controller reduction techniques such as those in [15] or frequency-weighted coprime factor reductions [12]. Alternatively, they can be achieved by noting that the proposed robust AW approach is especially practical for static or low-order AW approaches [11]. Indeed, choosing $Q(s)$ static and using the implementation in Figure 8, means that the number of states required are the same as for a standard full-order AW compensator, see [8].

[2]This is a standard requirement for even the non-robust anti-windup problem.

VII. Conclusions

A general framework for the design and analysis of robust anti-windup compensators has been presented. The proposed framework combines the W&P-AW scheme with a high-performance control architecture which includes a residual generator. This combination of schemes results in an analytical framework where the interactions between the different design parameters is transparent and which can be used to simplify the trade-off, during the synthesis stage, between robust stability and robust performance for the saturated and unsaturated closed-loops.

References

[1] Chen, J. and Patton, R.J. *Robust Model-Based Fault Diagnosis for Dynamic Systems*. Kluwer Academic Publishers, 1999.

[2] Crawshaw, S. and Vinnicombe, G. Anti-windup synthesis for guaranteed $\mathcal{L}_2$ performance. In *IEEE Conference on Decision and Control*, Sydney, Australia, December 2000.

[3] Ding, X. and Frank, P.M. Fault Detection via Factorization Approach. *Systems and Control Letters*, 14:431–436, 1990.

[4] Galeani, S., Nicosa, S., Teel, A.R., and Zaccarian, L. Output feedback compensators for weakened anti-windup of additively perturbed systems. In *IFAC World Congress*, Prague, July 2005.

[5] Garcia, G., Tarbouriech, S., Suarez, R., and Alvarez-Ramirez, J. Nonlinear bounded control for norm-bounded uncertain systems. *IEEE Transactions on Automatic Control*, 44(8):1254–1258, 1999.

[6] Hanus, R., Kinnaert, M., and Henrotte, J.L. Conditioning technique, a general anti-windup and bumpless transfer method. *Automatica*, 23(6):729–739, 1987.

[7] Kothare, M.V., Campo, P.J., Morari, M., and Nett, C.N. A unified framework for the study of anti-windup designs. *Automatica*, 30(12):1869–1883, 1994.

[8] Marcos, A., Turner, M.C., Bates, D.G., and Postlethwaite, I. Robustification of static and low order anti-windup designs. In *5th IFAC Symposium on Robust Control Design, ROCOND*, Toulouse, FR, July 2006.

[9] Tay, T.T., Mareels, I., and Moore, J.B. *High Performance Control*. Birkhauser, 1998.

[10] Turner, M.C., Herrmann, G., and Postlethwaite, I. Accounting for uncertainty in anti-windup synthesis. In *American Control Conference*, Boston, MA, June 2004.

[11] Turner, M.C. and Postlethwaite, I. A new perspective on static and low order anti-windup synthesis. *International Journal of Control*, 77(1):27–44, 2004.

[12] Varga, A. On frequency-weighted coprime factorization based controller reduction. In *American Control Conference*, Denver, USA, June 2003.

[13] Weston, P.F. and Postlethwaite, I. Linear conditioning for systems containing saturating actuators. *Automatica*, 36(8):1347–1354, August 2000.

[14] Zaccarian, L. and Teel, A. A common framework for anti-windup bumpless transfer and reliable design. *Automatica*, 38(10):1735–1744, October 2002.

[15] Zhou, K., Doyle, J.C., and Glover, K. *Robust and Optimal Control*. Prenticel-Hall, Englewood Cliffs, NJ, 1996.

[16] Zhou, K. and Ren, Z. A New Controller Architecture for High Performance, Robust, and Fault-Tolerant Control. *IEEE Transactions on Automatic Control*, 46(10):1613–1618, 2001.